Proceedings of the 38th
Conference on Decision & Control
Phoenix, Arizona USA • December 1999

Chained form transformation algorithm for a class of 3-states and 2-inputs nonholonomic systems and attitude control of a space robot

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Abstract

In this paper, we consider a chained form transformation for a class of the 3-states and 2-inputs symmetric affine nonholonomic systems. Firstly, we propose an algorithm for building the coordinate and input transformation to convert such systems into the chained form. It is shown that a chained form for a two-wheel car is easily obtained by using the transformation. Secondly, we show that some planar 3-link nonholonomic mechanical systems are expressed as the 3-states and 2-inputs symmetric affine system which we discuss. Finally, an asteroid sample return robot is considered as an example, and simulation has been carried out.

1 Introduction

It is known that the controller design for nonholonomic systems is difficult, because smooth static state feedback control can't ensure the asymptotic stability of the closed-loop system [1]. Some nonholonomic systems, for example free-flying space robots [2], [3] and legged robots [4], are expressed as symmetric affine systems. A chained form is a canonical form of symmetric affine systems and some control strategies based on it are proposed. If a symmetric affine nonholonomic system is transformed into the chained form, it could be comparatively easy to design a controller. However, in the algorithm [5] for building the transformation which accomplishes the conversion, we should solve partial differential equations and it is difficult to find the solutions in general.

In this paper, we consider a chained form transformation for a class of the 3-states and 2-inputs symmetric affine nonholonomic system. Firstly, we propose an algorithm for building the coordinate and input transformation to convert such systems into the chained form. It is shown that a chained form for a two-wheel car is easily obtained by using the transformation. Secondly, we show some examples which can be expressed as the 3-states and 2-inputs symmetric affine system. Finally, a space robot (an asteroid sample return robot) is considered and controllers are designed on the basis of the chained form. Simulation results are shown.

2 Chained Form Transformation

2.1 Algorithm for chained form transformation

Let us consider a 3-states and 2-inputs symmetric affine system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} a_{11}(x_1,x_2) & a_{12}(x_1,x_2) \\ a_{21}(x_1,x_2) & a_{22}(x_1,x_2) \\ a_{31}(x_1,x_2) & a_{32}(x_1,x_2) \end{bmatrix} \begin{bmatrix} \overline{u}_1 \\ \overline{u}_2 \end{bmatrix} \quad (1)$$

If the matrix

$$A(x_1,x_2) = \begin{bmatrix} a_{11}(x_1,x_2) & a_{12}(x_1,x_2) \\ a_{21}(x_1,x_2) & a_{22}(x_1,x_2) \end{bmatrix}$$

has its inverse, we introduce an input transformation

$$\begin{bmatrix} \overline{u}_1 \\ \overline{u}_2 \end{bmatrix} = \boldsymbol{A}^{-1}(x_1,x_2) \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

and (1) is rewritten as a 3-states and 2-inputs symmetric affine system

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \alpha(x_1,x_2) & \beta(x_1,x_2) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \\ &= \boldsymbol{g}_1(\boldsymbol{x})u_1 + \boldsymbol{g}_2(\boldsymbol{x})u_2 \end{aligned} \quad (2)$$

where

$$\begin{aligned} &[\alpha(x_1,x_2) \quad \beta(x_1,x_2)] \\ &= [a_{31}(x_1,x_2) \quad a_{32}(x_1,x_2)]\, \boldsymbol{A}^{-1}(x_1,x_2). \end{aligned}$$

The equation (2) can be regarded as a general form of the 3-states and 2-inputs symmetric affine systems (1).

We assume that the vector fields $\boldsymbol{g}_1(\boldsymbol{x}), \boldsymbol{g}_2(\boldsymbol{x})$ are smooth and independent. Let us define following three distributions

$$\begin{aligned} \Delta_0 &:= \mathrm{span}\{\boldsymbol{g}_1, \boldsymbol{g}_2, ad_{g_1}\boldsymbol{g}_2\}, \\ \Delta_1 &:= \mathrm{span}\{\boldsymbol{g}_2, ad_{g_1}\boldsymbol{g}_2\}, \\ \Delta_2 &:= \mathrm{span}\{\boldsymbol{g}_2\}. \end{aligned}$$

We assume that the system (2) satisfies the relation

$$\forall \boldsymbol{x} \in U \subseteq R^3 \quad s.t. \quad \dim \Delta_0 = 3$$

of the condition for the controllability. This relation can be rewritten as

$$\forall \boldsymbol{x} \in U \subseteq R^3 \quad s.t. \quad \frac{\partial \beta}{\partial x_1} - \frac{\partial \alpha}{\partial x_2} \neq 0. \tag{3}$$

Murray and Sastry [5] derived a set of sufficient conditions for determining if a controllable symmetric affine system can be converted to chained form under the following assumption.

$$\Delta_1, \Delta_2 \text{ is involutive on } U$$

And there exist the functions h_1, h_2 which satisfy the following conditions.

- $h_1, h_2 : \boldsymbol{R}^3 \longrightarrow \boldsymbol{R}$
- h_1 and h_2 are independent.
- $dh_1 \cdot \Delta_1 = 0$ and $dh_1 \cdot \boldsymbol{g}_1 = 1$ (4)
- $dh_2 \cdot \Delta_2 = 0$ and $dh_2 \cdot ad_{g_1} g_2 \neq 0$ (5)

If we could find the independent solutions h_1 and h_2 of the partial differential equations (4) and (5), the coordinate transformation $\boldsymbol{z} = \phi(\boldsymbol{x})$ and the input transformation $\boldsymbol{v} = \beta(\boldsymbol{x})\boldsymbol{u}$ to convert the system into the chained form

$$\dot{z}_1 = v_1 \ , \ \dot{z}_2 = v_2 \ , \ \dot{z}_3 = z_2 v_1 \tag{6}$$

can be expressed as

$$z_1 = h_1, \quad z_2 = L_{g_1} h_2, \quad z_3 = h_2 \tag{7}$$

$$v_1 := u_1, \quad v_2 := (L_{g_1}^2 h_2) u_1 + (L_{g_2} L_{g_1} h_2) u_2. \tag{8}$$

We should find solutions h_1, h_2, which satisfy (4),(5), for building the coordinate and input transformation to convert the system (2) into the chained form (6).

[Proposition]

The functions h_1, h_2 of solutions of the partial differential equations (4), (5) for building the coordinate and input transformation to convert the system (2) into the chained form (6) can be expressed as

$$h_1 = x_1 \tag{9}$$

$$h_2 = -\int \beta(x_1, x_2) dx_2 + x_3. \tag{10}$$

[Proof]

It is easy to find that h_1 and h_2 are independent. We see that the derived functions h_1, h_2 in (9),(10) satisfy the conditions (4),(5). Using (9),(10) gives

$$dh_1 = [1 \quad 0 \quad 0], \ dh_2 = [\star \quad -\beta(x_1, x_2) \quad 1] \tag{11}$$

and

$$dh_1 \cdot \boldsymbol{g}_2 = [1 \quad 0 \quad 0] \begin{bmatrix} 0 \\ 1 \\ \beta(x_1, x_2) \end{bmatrix} = 0$$

$$dh_1 \cdot ad_{g_1} g_2 = [1 \quad 0 \quad 0] \begin{bmatrix} 0 \\ 0 \\ \frac{\partial \beta}{\partial x_1} - \frac{\partial \alpha}{\partial x_2} \end{bmatrix} = 0$$

$$dh_1 \cdot \boldsymbol{g}_1 = [1 \quad 0 \quad 0] \begin{bmatrix} 1 \\ 0 \\ \alpha(x_1, x_2) \end{bmatrix} = 1$$

$$\begin{aligned} dh_2 \cdot \boldsymbol{g}_2 = dh_2 &= [\star \quad -\beta(x_1, x_2) \quad 1] \begin{bmatrix} 0 \\ 1 \\ \beta(x_1, x_2) \end{bmatrix} \\ &= -\beta(x_1, x_2) + \beta(x_1, x_2) = 0 \end{aligned}$$

$$\begin{aligned} dh_2 \cdot ad_{g_1} \boldsymbol{g}_2 &= [\star \quad -\beta(x_1, x_2) \quad 1] \begin{bmatrix} 0 \\ 0 \\ \frac{\partial \beta}{\partial x_1} - \frac{\partial \alpha}{\partial x_2} \end{bmatrix} \\ &= \frac{\partial \beta}{\partial x_1} - \frac{\partial \alpha}{\partial x_2} \neq 0. \end{aligned} \tag{12}$$

As from (3) the condition (12) is satisfied, we can find that the functions h_1, h_2 satisfy the condition (4),(5) and are the solutions of the partial differential equations. ∎

2.2 Example

Let us consider a two-wheel car as shown in Fig. 1.

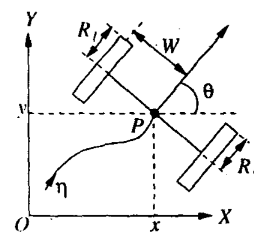

Fig. 1 Two-wheel car

The state equation is expressed as

$$\begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ 0 & 1 \\ \sin\theta & 0 \end{bmatrix} \begin{bmatrix} \overline{u}_1 \\ \overline{u}_2 \end{bmatrix}$$

$$\overline{u}_1 = \frac{d\eta}{dt} = \frac{R_l \omega_l + R_r \omega_r}{2}$$

$$\overline{u}_2 = \frac{d\theta}{dt} = \frac{-R_l \omega_l + R_r \omega_r}{2W}$$

where ω_l, ω_r are the angular velocities for the left wheel and the right wheel, respectively. Let us define the input transformation

$$\begin{bmatrix} \overline{u}_1 \\ \overline{u}_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{\cos\theta} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}.$$

Then we obtain the state equation

$$\begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \tan\theta & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}.$$

Using (9),(10) gives

$$h_1 = x \ , \ h_2 = y$$

and the coordinate transformation $\boldsymbol{z} = \phi(x, \theta, y)$ and the input transformation $\boldsymbol{v} = \beta(x, \theta, y)\boldsymbol{u}$ are obatined as

$$\begin{aligned} z_1 &= x, \ z_2 = L_{g_1} h_2 = \tan\theta, \ z_3 = y \\ v_1 &= u_1 = \cos\theta \overline{u}_1, \\ v_2 &= L_{g_1}^2 u_1 + L_{g_2} L_{g_1} u_2 = \sec^2\theta \overline{u}_2. \end{aligned}$$

Finally we obtain the chaind form (6).

3 Planar 3-link Mechanical Systems

We consider planar 3-link mechanical systems with prismatic and revolute joints. Let us introduce three assumptions

1. There is no external force.
2. The initial linear momentum of the system is zero. (13)
3. The initial angular momentum of the system is zero.

We consider four classes of the planar 3-link mechanical systems.

Type A : It has two revolute joints as shown in Fig. 2. The orientation of the system can be represented as the angle between the central axis of the first body and the X axis of the inertia coordinate.

$$x_1 = \theta_1, \ x_2 = \theta_2, \ x_3 = \varphi, \ u_1 = \dot{\theta}_1, \ u_2 = \dot{\theta}_2$$

$$\alpha(\theta_1, \theta_2) = \frac{b_1 + b_2\cos\theta_1 + b_3\cos\theta_2 + b_4\cos(\theta_1 + \theta_2)}{a_1 + a_2\cos\theta_1 + a_3\cos\theta_2 + a_4\cos(\theta_1 + \theta_2)}$$

$$\beta(\theta_1, \theta_2) = \frac{c_1 + c_2\cos\theta_2 + c_3\cos(\theta_1 + \theta_2)}{a_1 + a_2\cos\theta_1 + a_3\cos\theta_2 + a_4\cos(\theta_1 + \theta_2)}$$

Type B : It has two revolute joints as shown in Fig. 3. The orientation of the system can be represented as the angle between the central axis of the second body and the X axis of the inertia coordinate.

$$x_1 = \theta_1, \ x_2 = \theta_2, \ x_3 = \varphi, \ u_1 = \dot{\theta}_1, \ u_2 = \dot{\theta}_2$$

$$\alpha(\theta_1, \theta_2) = \frac{b_1 + b_2\sin\theta_1 + b_3\cos(\theta_1 - \theta_2)}{a_1 + a_2\sin\theta_1 + a_3\sin\theta_2 + a_4\cos(\theta_1 - \theta_2)}$$

$$\beta(\theta_1, \theta_2) = \frac{c_1 + c_2\sin\theta_2 + c_3\cos(\theta_1 - \theta_2)}{a_1 + a_2\sin\theta_1 + a_3\sin\theta_2 + a_4\cos(\theta_1 - \theta_2)}$$

Type C : It has the first revolute joint and the second prismatic joint as shown in Fig. 4. The orientation of the system can be represented as the angle between the central axis of the first body and the X axis of the inertia coordinate.

$$x_1 = \theta, \ x_2 = l, \ x_3 = \varphi, \ u_1 = \dot{\theta}, \ u_2 = \dot{l}$$

$$\alpha(\theta, l) = \frac{b_1 l^2 + b_2 l + b_3 l\cos\theta + b_4}{a_1 l^2 + a_2 l + a_3 l\cos\theta + a_4}$$

$$\beta(\theta, l) = \frac{c_1\sin\theta}{a_1 l^2 + a_2 l + a_3 l\cos\theta + a_4}$$

Type D : It has the first prismatic joint and the second revolute joint as shown in Fig. 5. The orientation of the system can be represented as the angle between the central axis of the first and second body and the X axis of the inertia coordinate.

$$x_1 = \theta, \ x_2 = l, \ x_3 = \varphi, \ u_1 = \dot{\theta}, \ u_2 = \dot{l}$$

$$\alpha(\theta, l) = \frac{b_1 l\cos\theta + b_2}{a_1 l^2 + a_2 l + a_3 l\cos\theta + a_4}$$

$$\beta(\theta, l) = \frac{c_1\sin\theta}{a_1 l^2 + a_2 l + a_3 l\cos\theta + a_4}$$

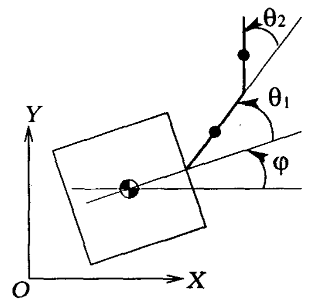

Fig. 2 Planar 3-link mechanical system with two revolute joints (Type A)

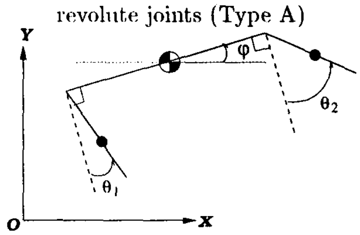

Fig. 3 Planar 3-link mechanical system with two revolute joints (Type B)

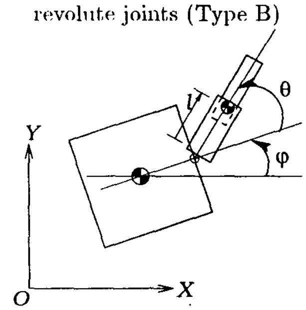

Fig. 4 Planar 3-link mechanical system with revolute and prismatic joints (Type C)

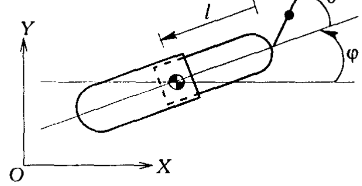

Fig. 5 Planar 3-link mechanical system with prismatic and revolute joints (Type D)

We find that models of the planar 3-link mechanical systems under the assumption (13) can be expressed as the form (2). The planar 3-link mechanical systems which has two prismatic joints are neglected, because the mechanisum does not have any merit.

4 Attitude Control of an Asteroid Sample Return Robot

First we explain the outline of the asteroid sample return (MUSES-C) mission. The primary goal of the MUSES-C mission is to acquire and verify technology which is necessary to return samples from a small body in the solar system and bring back them to the earth. We consider an asteroid sample return robot without attitude control devices for example reaction wheels and jets.

The sample return robot is released from the space shuttle to the asteroid with a constant linear momentum. The center of mass of the whole sample return robot approaches to the asteroid with the constant velocity. In the approaching phase the configuration of the manipulator and the attitude of the robot body should be controlled by actuating only manipulator joints.

In the legged robot which is treated by Li and Montgomery [4] and the space robot which is treated by Nakagawa et al. [6], the revolute joint of a body is located at the center of mass of the base body. These examples are easy to treat and special cases of the robot system that we consider.

In this section we derive a mathematical model by using the conservation law of linear and angular momentum. By applying the proposed functions the system is transformed into the cahined form. On the basis of the cahined form open-loop and closed-loop controllers are designed.

4.1 Model

Let us consider the sample return robot in the planar sapce as shown in Fig. 6. The robot is the Type C mechanical system in the section 3. Let us define the inertial coordinate frame $O-XY$, the center of mass of the whole system P_{CM}, the base body P_0, the arm P_1. Let us define position vectors $\boldsymbol{r}_{CM} = \overrightarrow{OP}_{CM}$ $\boldsymbol{r}_0 = \overrightarrow{OP}_0$, $\boldsymbol{r}_1 = \overrightarrow{OP}_1$. The direction of X axis is same as the velocity vector $\dot{\boldsymbol{r}}_{CM}$. Let φ and θ be the attitude angle of the base body and the joint angle of the arm. Let us define $l(t)$ and d be the distances from the rotational center of the revolute joint to P_1 and to P_0, respectively. Let m_0, m_1 be the mass of the base body and the arm, and $M(=m_0+m_1)$ be the total mass, respectively. Let I_0 and $I_1(l)$ be the moment of inertia of the base body and the arm about the center of mass. Let d_1 be the distance from the rotational center of the revolute joint of the first link to the center of mass of the first link. Let I_{1i}, m_{1i} be the moment of inertia and the mass of the link i. The moment of inertia of the arm can be expressed as $I_1(l) = I_{11} + I_{12} + \frac{m_1 m_{11}}{m_{12}}(l-d_1)^2$. Assuming constant linear and zero angular momentum of the system at the initial time, the linear and angular momentum conservation equations are expressed as

$$m_0\dot{\boldsymbol{r}}_0 + m_1\dot{\boldsymbol{r}}_1 = \boldsymbol{C}, \qquad (14)$$
$$I_0\dot{\varphi} + I_1(l)(\dot{\theta}+\dot{\varphi}) + m_0\boldsymbol{r}_0\times\dot{\boldsymbol{r}}_0 + m_1\boldsymbol{r}_1\times\dot{\boldsymbol{r}}_1 = 0$$

where $\boldsymbol{C}$ is a constant vector. Let us define the state and input vectors as

$$\boldsymbol{x} = [\,\theta-\theta_d \quad l-l_d \quad \varphi-\varphi_d\,]^T, \quad \boldsymbol{u} = [\,\dot{\theta} \quad \dot{l}\,]^T$$

where θ_d, l_d, φ_d are the desired values. From the equations (14) the state equation can be represented as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\\ \alpha(x_1,x_2) & \beta(x_1,x_2)\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix} \qquad (15)$$

where

$$\begin{aligned}
\alpha(x_1,x_2) &= f_1(x_1+\theta_d, x_2+l_d)\\
\beta(x_1,x_2) &= f_2(x_1+\theta_d, x_2+l_d)\\
f_1(\theta,l) &= -\frac{Bl^2+(-2q+p\cos\theta)l+D}{Bl^2+(-2q+2p\cos\theta)l+C}\\
f_2(\theta,l) &= -\frac{p\sin\theta}{Bl^2+(-2q+2p\cos\theta)l+C}\\
B &= \frac{m_1 m_{11}}{m_{12}} + \frac{m_0 m_1}{M}\\
C &= I_0 + I_{11} + I_{12} + \frac{d^2 m_0 m_1}{M} + \frac{d_1^2 m_1 m_{11}}{m_{12}}\\
D &= I_{11} + I_{12} + \frac{d_1^2 m_1 m_{11}}{m_{12}}\\
p &= \frac{d m_0 m_1}{M}, \; q = \frac{d_1 m_1 m_{11}}{m_{12}}.
\end{aligned}$$

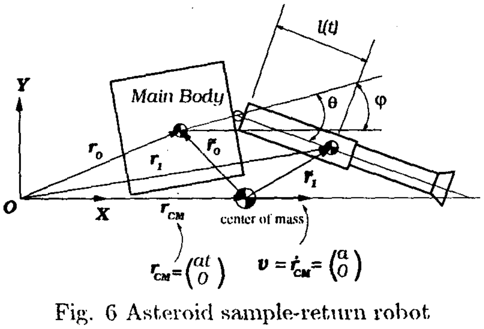

Fig. 6 Asteroid sample-return robot

4.2 Chained form transformation

Using the proposition we can obtain the functions

$$h_1 = x_1 \qquad (16)$$

$$h_2 = \frac{\sqrt{2}p\sin(x_1+\theta_d)\arctan\left[\frac{\sqrt{2}S(x_1,x_2)}{\sqrt{U(x_1)}}\right]}{\sqrt{U(x_1)}} + x_3 \qquad (17)$$

where

$$\begin{aligned} S(x_1,x_2) &= -q+p\cos(x_1+\theta_d)+B(x_2+l_d) \\ U(x_1) &= 2BC-p^2-2p^2+4pq\cos(x_1+\theta_d) \\ &\quad -p^2\cos(2(x_1+\theta_d)). \end{aligned}$$

The coordinate and input transformation is obtained as

$$\begin{aligned} z_1 &= x_1 \\ z_2 &= \left(\frac{\sqrt{2}p\cos(x_1+\theta_d)}{\sqrt{U(x_1)}} - \frac{p\sin(x_1+\theta_d)T(x_1)}{\sqrt{2}U^{\frac{3}{2}}(x_1)}\right) \\ &\times \arctan\left[\frac{\sqrt{2}S(x_1,x_2)}{\sqrt{U(x_1)}}\right] - \frac{\sqrt{2}p\sin(x_1+\theta_d)}{U(x_1)+2S^2(x_1,x_2)} \\ &\times \left(\sqrt{2}p\sin(x_1+\theta_d)+\frac{S(x_1,x_2)T(x_1)}{\sqrt{2}U(x_1)}\right)+\alpha(x_1,x_2) \\ z_3 &= \frac{\sqrt{2}p\sin(x_1+\theta_d)\arctan\left[\frac{\sqrt{2}S(x_1,x_2)}{\sqrt{U(x_1)}}\right]}{\sqrt{U(x_1)}}+x_3 \\ v_1 &= u_1 \\ v_2 &= \frac{\partial z_2}{\partial x_1}u_1+\frac{\partial z_2}{\partial x_2}u_2. \end{aligned} \quad (18)$$

4.3 Control law and simulation

The physical parameters of the robot are taken as shown in Table 1. The initial configuration in the open-loop and closed-loop control are taken as $x_1^0 = \frac{\pi}{2}$[rad], $x_2^0 = -0.5$[m], $x_3^0 = 0$[rad] and $x_1^0 = 0$[rad], $x_2^0 = 0$[m], $x_3^0 = \frac{\pi}{12}$[rad], respectively. The desired configuration is taken as $x_1^d = 0$[rad], $x_2^d = 0$[m], $x_3^d = 0$[rad]. Because of the limitation of the extent of movement a constraint condition should be considered. The state constraint condition

$$-\frac{\pi}{2} \le x_1(t) \le \frac{\pi}{2}\ ,\ -0.5 \le x_2(t) \le 0 \quad (19)$$

is considered.

Table 1 Parameters of robot

	Base body	Arm 1	Arm 2
Weight [kg]	300	30	30
Size [m]	2×2	1×0.3	1×0.3
Moment of inertia [kgm^2]	200	2.725	2.725

We design the open-loop controller based on sinusoids [5] and the closed-loop controller based on the time-state control form [7].

In the open-loop control the input is given as

$$\begin{aligned} &0[\text{sec}] \le\ t\ \le 5[\text{sec}] \\ &u_1(t) = -\frac{\pi}{10},\ u_2(t)=0.1, \\ &5[\text{sec}] \le\ t\ \le 30[\text{sec}] \\ &v_1(t) = 0.4615\cos\left(\frac{2\pi}{5}(t-5)\right) \\ &v_2(t) = 0.0518\sin\left(\frac{2\pi}{5}(t-5)\right). \end{aligned}$$

In the closed-loop control we introduce the input transformation from the chained form (6) to the time-state control form.

$$\mu_1 = v_1,\ \mu_2 = \frac{v_2}{v_1}$$

From the transformation we obtain the time-state control form

$$\frac{d}{dz_1}\begin{bmatrix} z_3 \\ z_2 \end{bmatrix} = \begin{bmatrix} z_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\mu_2 \quad (20)$$

$$\frac{dz_1}{dt} = \mu_1. \quad (21)$$

As the obtained system (20) is linear, the feedback control

$$\mu_2 = -k_2z_2 - k_3z_3 \quad (22)$$

can stabilize the system. The input v_2 for the chained form is given as

$$v_2(t) = \begin{cases} -k_2z_2v_1 - k_3z_3v_1,\ v_1 > 0\ \ (\dot{z}_1 > 0) \\ k_2z_2v_1 - k_3z_3v_1,\ v_1 < 0\ \ (\dot{z}_1 < 0). \end{cases} \quad (23)$$

In the simulation the feedback gains are taken as $k_2 = 8, k_3 = 16$ and the input $v_1(t) = \mu_1(t)$ is given as

$$v_1(t) = \begin{cases} \frac{\pi}{10} & \text{if } 0 \le t < 5, 15 \le t \le 25 \\ -\frac{\pi}{10} & \text{if } 5 \le t < 15. \end{cases}$$

Figs. 7 and 8 show the transisent responses for the open-loop control and the closed-loop control, respectively. In the open-loop control it is easy to obtain the control input for big initial errors with considering the constraint condition, but the controller is not robust for disturbances. In the closed-loop control the controller is robust for the disturbances, but the applicability of it is restricted because of the state constraint. The calculation for the inverse transformation from the input for the chained form to the original input is complicated.

5 Concluding Remarks

We have considered a chained form transformation for a class of the 3-states and 2-inputs symmetric affine nonholonomic system. The solutions of the partial differential equations are obtained for building the coordinate and input transformation to convert the system into the chained form. An asteroid sample return robot is considered as an example, and simulation has been carried out. We should consider following problems : 1). Robust controller design for parameter uncertainty, 2). Controller design for the nonholonomic system, with the drift term (for example, a space robot with non-zero initial angular momentum) 3). Development of the chained form transformation algorithm for wider calss of the nonholonomic systems.

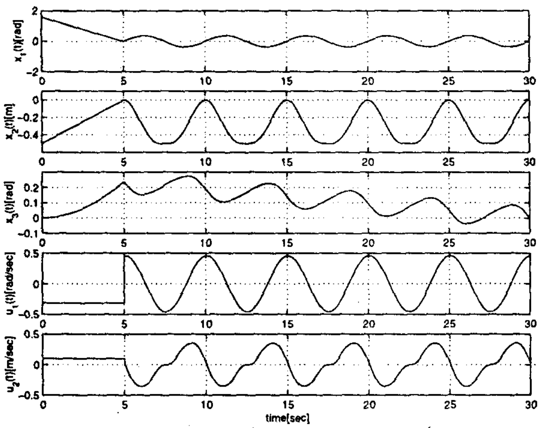

Fig. 7 Transient responses for open-loop controller

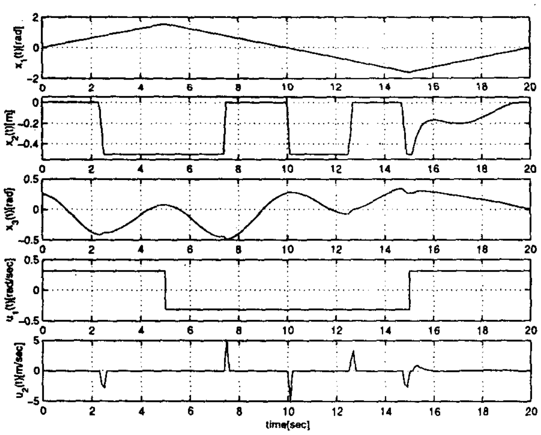

Fig. 8 Transient responses for closed-loop controller

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A Appendix

The definition of the function in the coordinate and input transformation in (18).

$$T(x_1) = -4pq\sin(x_1+\theta_d) + 2p^2\sin(2(x_1+\theta_d))$$
$$V(x_1) = -4pq\cos(x_1+\theta_d) + 4p^2\cos(2(x_1+\theta_d))$$
$$W(x_1,x_2) = U(x_1) + 2S^2(x_1,x_2)$$

$$\frac{\partial z_2}{\partial x_1} = \sum_{i=1}^{4}\xi_i \arctan\left(\frac{\sqrt{2}S(x_1,x_2)}{\sqrt{U(x_1)}}\right) + \sum_{i=5}^{8}\xi_i - \frac{p(x_2+l_d)\sin(x_1+\theta_d)(B(x_2+l_d)^2 - 2q(x_2+l_d) + 2D - C)}{(B(x_2+l_d)^2 + (-2q+2p\cos(x_1+\theta_d))(x_2+l_d) + C)^2}$$

$$\begin{aligned}\frac{\partial z_2}{\partial x_2} = {} & \frac{2pB}{W(x_1,x_2)}\left(\cos(x_1+\theta_d) - \frac{\sin(x_1+\theta_d)T(x_1)}{U(x_1)}\right) \\ & + \frac{4pB\sin(x_1+\theta_d)S(x_1,x_2)}{W^2(x_1,x_2)} \times \left(2p\sin(x_1+\theta_d) + \frac{S(x_1,x_2)T(x_1)}{U(x_1)}\right) \\ & + \frac{2S(x_1,x_2)\alpha(x_1,x_2) - 2B(x_2+l_d) - 2q + p\cos(x_1+\theta_d)}{B(x_2+l_d)^2 + (-2q+2p\cos(x_1+\theta_d))(x_2+l_d) + C}\end{aligned}$$

$$\xi_1 = -\frac{\sqrt{2}p\sin(x_1+\theta_d)}{\sqrt{U(x_1)}}, \quad \xi_2 = -\frac{p\sin(x_1+\theta_d)V(x_1)}{\sqrt{2}U^{\frac{3}{2}}(x_1)}$$

$$\xi_3 = -\frac{\sqrt{2}p\cos(x_1+\theta_d)T(x_1)}{\sqrt{U(x_1)}}$$

$$\xi_4 = \frac{3p\sin(x_1+\theta_d)T^2(x_1)}{2\sqrt{2}U^{\frac{5}{2}}(x_1)}$$

$$\xi_5 = -\frac{2p\cos(x_1+\theta_d)}{W(x_1,x_2)} \times \left(2p\sin(x_1+\theta_d) + \frac{S(x_1,x_2)T(x_1)}{U(x_1)}\right)$$

$$\xi_6 = \frac{p\sin(x_1+\theta_d)T(x_1)}{U(x_1)W(x_1,x_2)} \times \left(2p\sin(x_1+\theta_d) + \frac{S(x_1,x_2)T(x_1)}{U(x_1)}\right)$$

$$\xi_7 = \frac{2p\sin(x_1+\theta_d)S(x_1,x_2)}{U(x_1)W(x_1,x_2)} \times \left(2p\sin(x_1+\theta_d) + \frac{S(x_1,x_2)T(x_1)}{U(x_1)}\right)$$

$$\xi_8 = \frac{p\sin(x_1+\theta_d)}{W(x_1,x_2)} \times \left(-2p\cos(x_1+\theta_d) - \frac{S(x_1,x_2)V(x_1)}{U(x_1)} + \frac{2p\sin(x_1+\theta_d)T(x_1)}{U(x_1)} + \frac{3S(x_1,x_2)T^2(x_1)}{2U^2(x_1)}\right)$$