

Modeling of Unmanned Aerial Vehicles

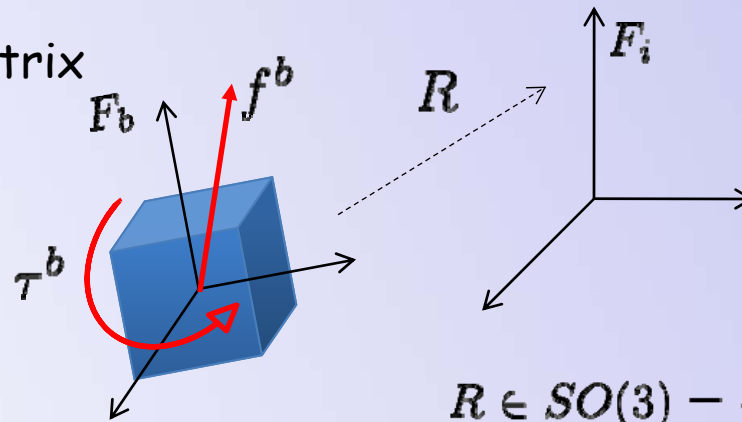
- Modeling in free-flight
 - Rigid body
 - Force/Torque generation mechanisms
 - Quadrotors, Helicopters
 - Ducted-fan, Coaxial (postponed)
 - Control of under-actuated vehicles
 - Inner-outer loop strategy
 - The linearized system
 - UNIBO research activity
- Modeling in presence of contact (R. Naldi)

Rigid Body



- Newton-Euler equations of motion

R Rotation matrix
 M Mass
 J Inertia



$$\vec{p} = p^i = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\vec{\omega} = \omega^b = \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$

$$R \in SO(3) = \{R \in \mathbb{R}^{3 \times 3} : R^T R = R R^T = I, \det(R) = 1\}$$

$$\begin{aligned} M \ddot{\vec{p}} &= R f^b \\ \dot{R} &= R \text{Skew}(\vec{\omega}) \\ J \dot{\vec{\omega}} &= -\text{Skew}(\vec{\omega}) J \vec{\omega} + \tau^b \end{aligned}$$

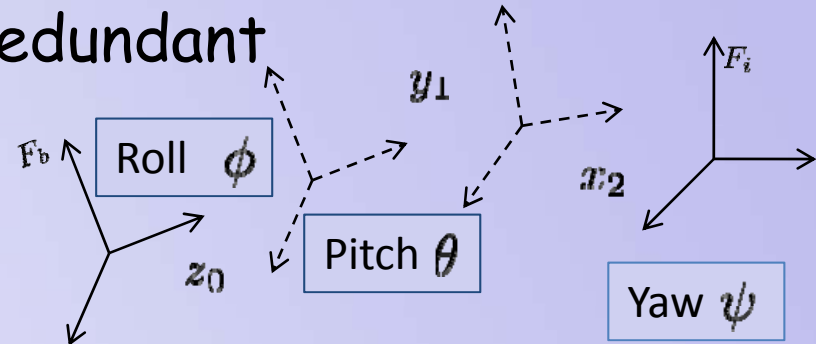
$$(\vec{p}, R) \in SE(3) = \mathbb{R}^3 \times SO(3)$$

$$\vec{v} \times \vec{w} = \text{Skew}(\vec{v})\vec{w} \quad \text{Skew}(\vec{v}) = \begin{pmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{pmatrix}$$

Rigid Body



- R computationally nice but redundant
- Attitude parameterization
 - Roll/Pitch/Yaw



$$\Theta := \begin{pmatrix} \phi \\ \theta \end{pmatrix} \quad \Theta_\psi := \begin{pmatrix} \Theta \\ \psi \end{pmatrix}$$

$$R(\Theta_\psi) = \begin{pmatrix} C_\psi C_\theta & -S_\psi C_\phi + C_\psi S_\theta S_\phi & S_\phi S_\psi + C_\phi S_\theta C_\psi \\ S_\psi C_\theta & C_\phi C_\psi + S_\phi S_\theta S_\psi & -C_\psi S_\phi + S_\psi S_\theta C_\phi \\ -S_\theta & C_\theta S_\phi & C_\theta C_\phi \end{pmatrix}$$

$$\dot{R} = R \text{Skew}(\vec{\omega})$$



$$\dot{\Theta}_\psi = Q(\Theta)\vec{\omega} \quad Q(\Theta) = \begin{pmatrix} 1 & S_\phi T_\theta & C_\phi T_\theta \\ 0 & C_\phi & -S_\phi \\ 0 & S_\phi / C_\theta & C_\phi / C_\theta \end{pmatrix}.$$

Minimal ($\Theta_\psi \in \mathbb{R}^3$) but containing singularities

- Attitude parameterization

– Quaternions $\mathbf{q} = \begin{pmatrix} q_0 \\ \mathbf{q} \end{pmatrix} \in S_4 := \{x \in \mathbb{R}^4 : \|x\| = 1\}$ $q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$

$$R(\mathbf{q}) = \begin{pmatrix} 1 - 2q_2^2 - 2q_3^2 & 2q_1q_2 - 2q_0q_3 & 2q_1q_3 + 2q_0q_2 \\ 2q_1q_2 + 2q_0q_3 & 1 - 2q_1^2 - 2q_3^2 & 2q_2q_3 - 2q_0q_1 \\ 2q_1q_3 - 2q_0q_2 & 2q_2q_3 + 2q_0q_1 & 1 - 2q_1^2 - 2q_2^2 \end{pmatrix}$$

- $R(\mathbf{q}) \in SO(3)$
- $\forall R^* \in SO(3)$
 $\exists \mathbf{q}^* \in S_4$
s.t. $R(\mathbf{q}^*) = R^*$

$$\dot{R} = R \text{Skew}(\vec{\omega})$$



$$\dot{\mathbf{q}} = \frac{1}{2} E(\mathbf{q}) \vec{\omega}, \quad E(\mathbf{q}) = \begin{pmatrix} -\mathbf{q}^T \\ q_0 I + \text{Skew}(\mathbf{q}) \end{pmatrix}$$

Non minimal ($\mathbf{q} \in \mathbb{R}^4$) but
non singular and ...
nice algebra

Rigid Body



- Overall

$$\begin{aligned}
 M\ddot{\vec{p}} &= R f^b \\
 \dot{R} &= R \text{Skew}(\vec{\omega}) \\
 J\dot{\vec{\omega}} &= -\text{Skew}(\vec{\omega}) J \vec{\omega} + \tau^b
 \end{aligned}
 \quad \mathbb{R}^6 \times SO(3) \times \mathbb{R}^3$$

- rpy

$$\begin{aligned}
 M\ddot{\vec{p}} &= R(\Theta_\psi) f^b \\
 \dot{\Theta}_\psi &= Q(\Theta) \vec{\omega} \\
 J\dot{\vec{\omega}} &= -\text{Skew}(\omega) J \vec{\omega} + \tau^b
 \end{aligned}$$

$$\mathbb{R}^6 \times \mathbb{R}^3 \times \mathbb{R}^3$$

- quaternions

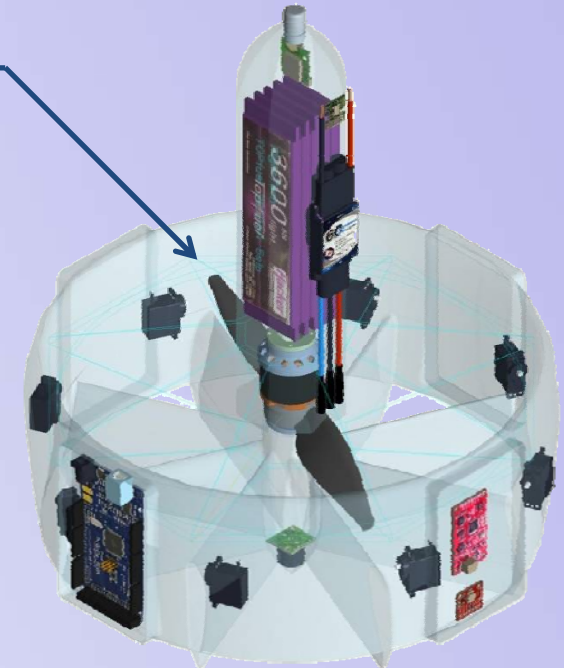
$$\begin{aligned}
 M\ddot{\vec{p}} &= R(\mathbf{q}) f^b \\
 \dot{\mathbf{q}} &= \frac{1}{2} E(\mathbf{q}) \vec{\omega} \\
 J\dot{\vec{\omega}} &= -\text{Skew}(\omega) J \vec{\omega} + \tau^b
 \end{aligned}$$

$$\mathbb{R}^6 \times S_4 \times \mathbb{R}^3$$

(f^b, τ^b) wrench vector

- Ducted-fan

- ✓ A fixed pitch propeller powered by electric motor
 - Ducted fan structure to
 - Protect the environment from moving parts
 - Improve the efficiency of the propeller
- ✓ A set of actuated control surfaces
 - Profiled surfaces driven by a servo controller

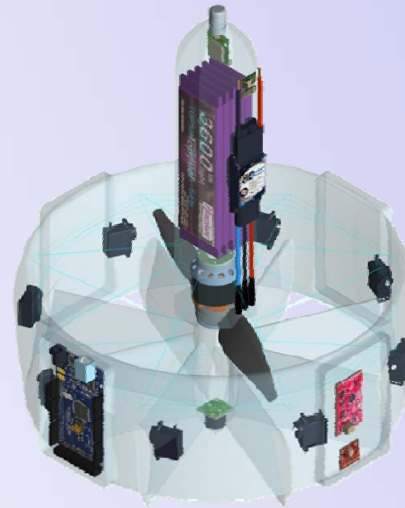


- **Vanes architectures** (3 dof, 3 motors in principle)

- Single level,
"minimalistic" conf.



- Single level,
"redundant" conf.



Control allocation policies

- Two levels,
Yaw, Roll-Pitch



- Ducted-fan literature:

- Project AROD <http://www.dtic.mil/dtic/tr/fulltext/u2/a271957.pdf>
- Caltech Ducted-Fan [A. Jadbabaie et al., 1999] , [J. Yu et al, 1999]
- iSTAR Micro Air Vehicle [L. Lipera et al, 2001], [I. Guerrero et al, 2001] , Allied Aerospace
- GTSpy Ducted-Fan [E. N. Johnson and M. A. Turbe, 2004] Georgia tech
- "Hovereye" [Pflimlin et al., 2004] (supported by BERTIN tech)
- ...

- Modeling the propeller thrust

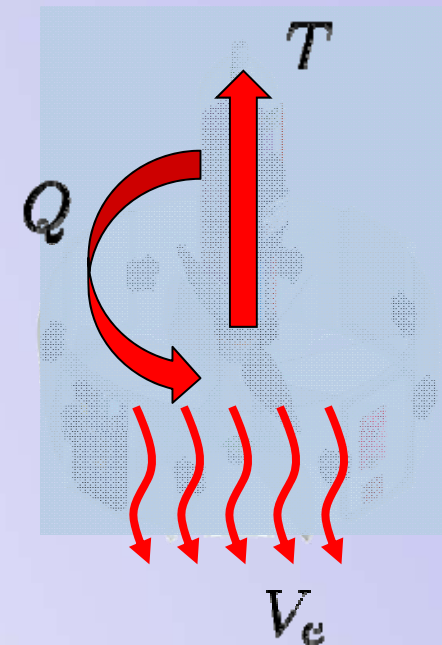
- Two theories available to model a propeller disk:
 - Actuator disk theory (based on Bernoulli equations)
 - Blade element theory

- Thrust: $T \approx K_T w_e^2$

- Reaction torque: $Q \approx K_Q w_e^2$

- Downwash: $V_e \approx K_V w_e$

- Propeller speed: w_e

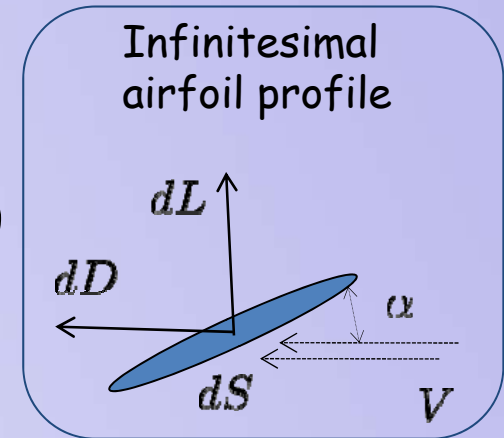


Wrench generation mechanism

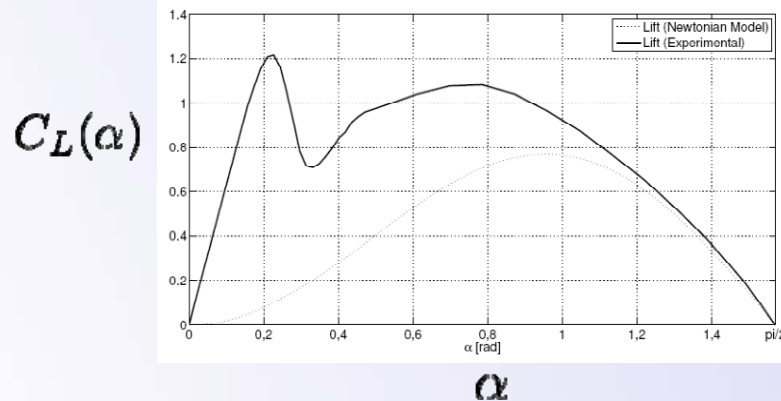


- Basic aerodynamic principles
 - Drag force (direction of the relative wind)
 - Lift force (perpendicular to the relative wind)

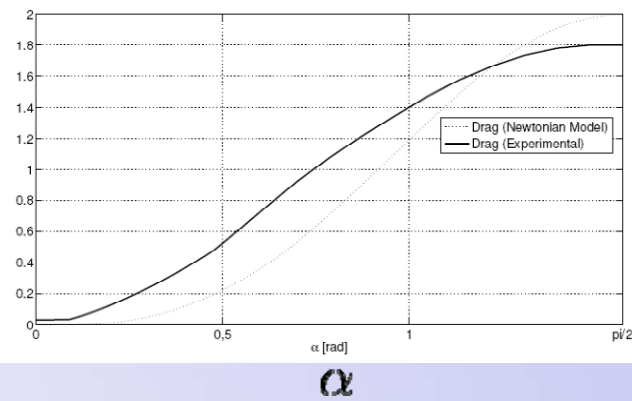
$$dL = \frac{1}{2} \rho V^2 C_L dS \quad dD = \frac{1}{2} \rho V^2 C_D dS$$



- Lift and drag coefficients $C_L(\alpha)$, $C_D(\alpha)$



$$C_L(\alpha) \approx K_L \alpha$$



$$C_D(\alpha) \approx K_D \alpha^2 + C_{D0}$$

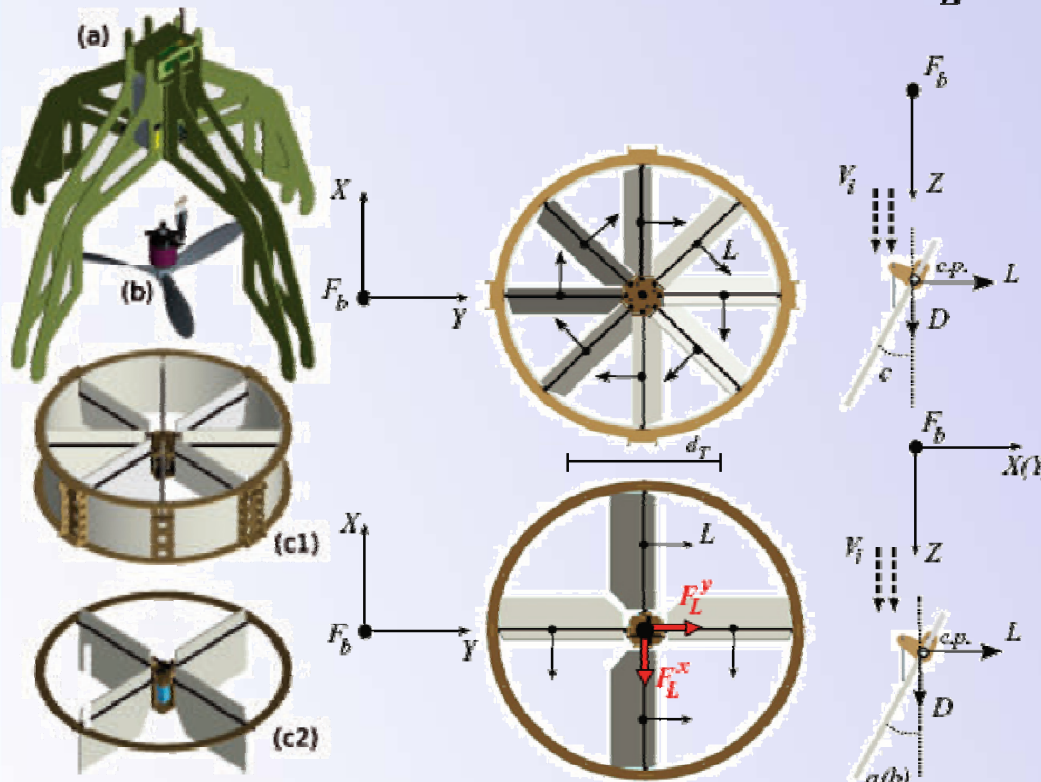
Wrench generation mechanism



- Aerodynamic forces

$$L = \frac{1}{2} \rho V_e^2 C_L S \quad D = \frac{1}{2} \rho V_e^2 C_D S$$

$$C_L \approx K_L \alpha \quad C_D \approx K_D \alpha^2 + C_{D0}$$



$$f^b = \begin{pmatrix} F_L^x \\ F_L^y \\ -T + F_D \end{pmatrix} + R^T \begin{pmatrix} 0 \\ 0 \\ Mg \end{pmatrix}$$

$$F_L^x = \kappa_x V_e^2 a \quad F_L^y = \kappa_y V_e^2 b$$

$$V_e = K_{V_e} w_e^2 \quad T = K_T w_e^2$$

F_D sum of the drag forces

$$\tau^b = \begin{pmatrix} -F_L^y d \\ F_L^x d \\ Q + Q_c \end{pmatrix} + R^T w_e G \vec{\omega}$$

$$Q = K_Q w_e^2 \quad Q_c = \kappa_z V_e^2 c$$

$R^T w_e G \vec{\omega}$ Gyroscopic precession propeller

$$G = \text{Skew}([0, 0, I_{\text{rot}}])$$

Side forces (often neglected)

$$f^b = w_e^2 \begin{pmatrix} \epsilon_x a \\ \epsilon_y b \\ -K_T + F'_D \end{pmatrix} + R^T \begin{pmatrix} 0 \\ 0 \\ Mg \end{pmatrix} \quad f^b \approx w_e^2 \begin{pmatrix} 0 \\ 0 \\ -K_T \end{pmatrix} + R^T \begin{pmatrix} 0 \\ 0 \\ Mg \end{pmatrix}$$

Drag forces (often neglected)

$$\tau^b = w_e^2 \begin{pmatrix} 0 & -a_1 & 0 \\ a_1 & 0 & 0 \\ 0 & 0 & a_2 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} + w_e^2 \begin{pmatrix} 0 \\ 0 \\ K_Q \end{pmatrix} + R^T w_e G \bar{w}$$

Aerodynamic drag forces neglected

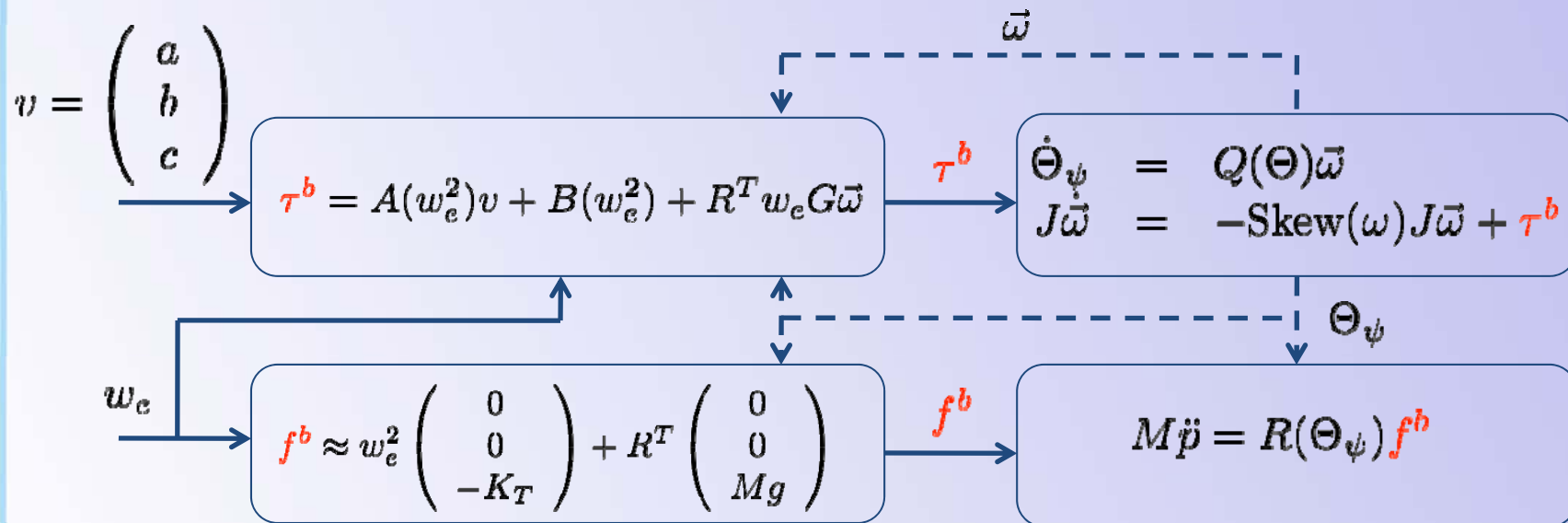
Side forces relevant in output feedback (zero dynamics)

Wrench generation mechanism

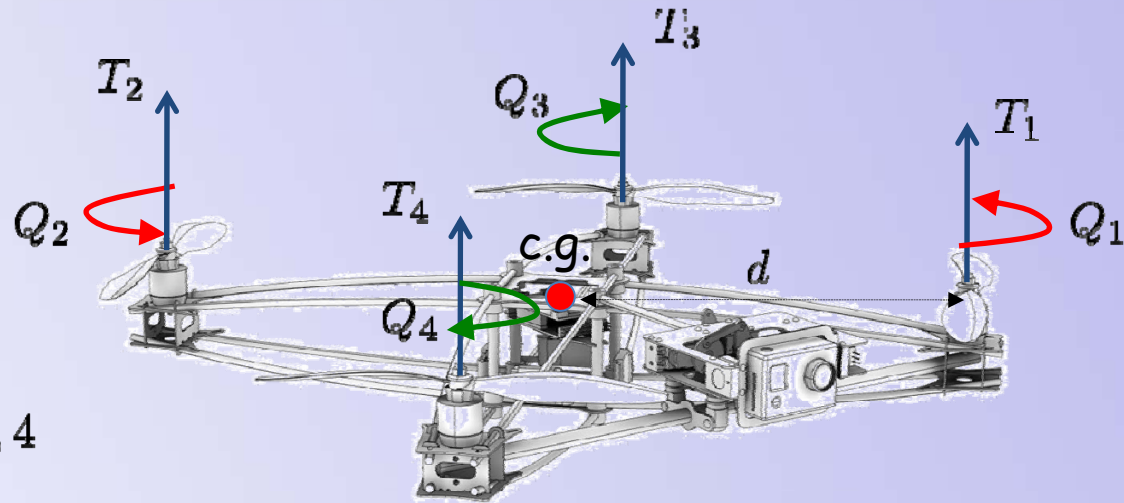
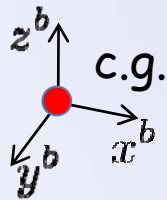


$$f^b \approx w_e^2 \begin{pmatrix} 0 \\ 0 \\ -K_T \end{pmatrix} + R^T \begin{pmatrix} 0 \\ 0 \\ Mg \end{pmatrix}$$

$$\tau^b = w_e^2 \begin{pmatrix} 0 & -a_1 & 0 \\ a_1 & 0 & 0 \\ 0 & 0 & a_2 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} + w_e^2 \begin{pmatrix} 0 \\ 0 \\ K_Q \end{pmatrix} + R^T w_e G \bar{w}$$



- Quadrotor



$$Q_i = K_Q w_{ei}^2, \quad i = 1, \dots, 4$$

$$T_i = K_T w_{ei}^2, \quad i = 1, \dots, 4$$

$$w_e^2 = \sum_{i=1}^4 w_{ei}^2$$

$$a = (w_{e3}^2 - w_{e4}^2) / w_e^2$$

$$b = (w_{e1}^2 - w_{e2}^2) / w_e^2$$

$$c = (w_{e2}^2 - w_{e3}^2) / w_e^2$$

$$f^b = \begin{pmatrix} 0 \\ 0 \\ \sum_{i=1}^4 T_i \end{pmatrix} + R^T \begin{pmatrix} 0 \\ 0 \\ Mg \end{pmatrix} \quad \tau^b = \begin{pmatrix} T_3 d - T_4 d \\ T_1 d - T_2 d \\ Q_1 + Q_2 - Q_3 - Q_4 \end{pmatrix}$$

$$f^b = w_e^2 \begin{pmatrix} 0 \\ 0 \\ K_T \end{pmatrix} + R^T \begin{pmatrix} 0 \\ 0 \\ Mg \end{pmatrix}$$

$$\tau^b = w_e^2 \begin{pmatrix} K_T d & 0 & 0 \\ 0 & K_T d & 0 \\ K_Q & K_Q & -2K_Q \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

- Helicopter (Gravilets, 2003)

$$f^b = \begin{pmatrix} X_M \\ Y_M + Y_T \\ Z_M \end{pmatrix} + R^T \begin{pmatrix} 0 \\ 0 \\ Mg \end{pmatrix}$$

$$\tau^b = \begin{pmatrix} \tau_{f1} \\ \tau_{f2} \\ \tau_{f3} \end{pmatrix} + \begin{pmatrix} R_M \\ M_M \\ N_M \end{pmatrix}$$

$$X_M = -T_M \sin a \quad Y_M = T_M \sin b,$$

$$Z_M = -T_M \cos a \cos b \quad Y_T = -T_T$$

$$\tau_{f1} = Y_M h_M + Z_M y_M + Y_T h_T$$

$$\tau_{f2} = -X_M h_M + Z_M \ell_M$$

$$\tau_{f3} = -Y_M \ell_M - Y_T \ell_T$$

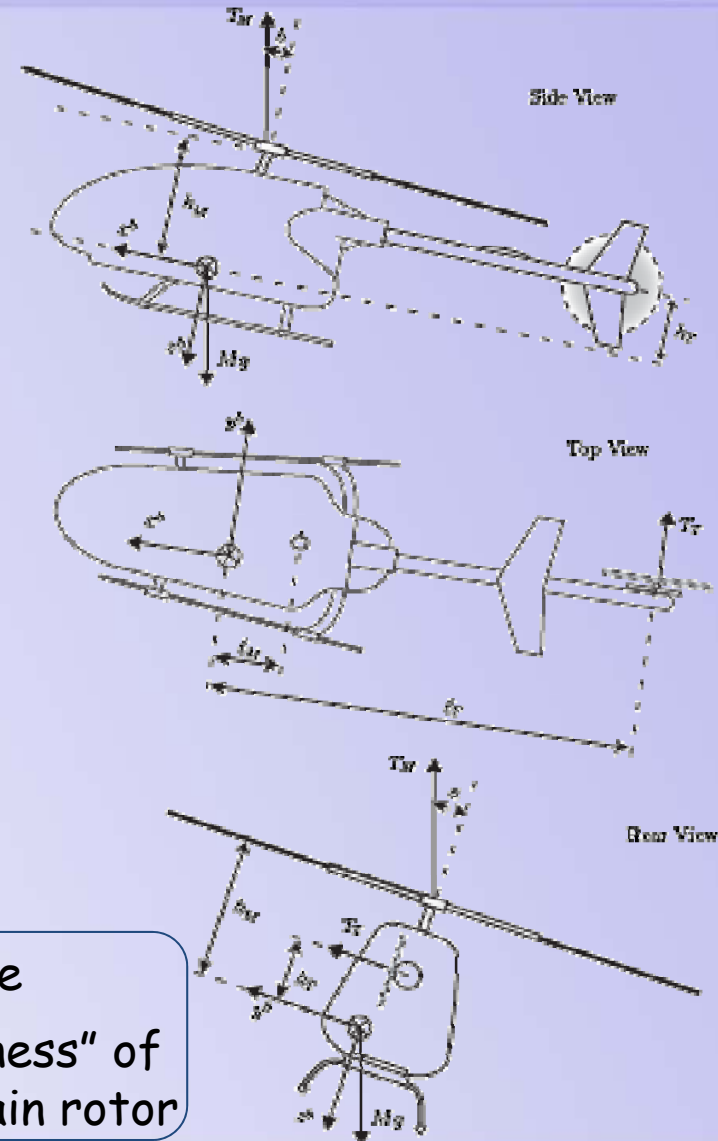
$$R_M = c_b^M b - Q_M \sin a$$

$$M_M = c_a^M a + Q_M \sin b$$

$$N_M = -Q_M \cos a \cos b$$

Q_M rotor torque

(C_a^M, C_b^M) "stiffness" of the main rotor



- **Helicopter** (a, b) Lateral/longitudinal inclination of the tip path plane of the main rotor (cyclic pitch)

$$\left. \begin{aligned} T_M &= K_{T_M} P_M w_e^2 \\ T_T &= K_{T_T} P_T w_e^2 \end{aligned} \right\} (P_M, P_T) \text{ Collective main/tail rotor pitches}$$

$$J_{\text{rot}} \dot{w}_e = k_e T_h + Q_M(w_e, P_M) \quad T_h \text{ Throttle of the main engine}$$

$$\text{Control inputs: } v = \begin{pmatrix} a \\ b \\ P_T \end{pmatrix}, P_M, \text{ and } T_h \text{ (to control } w_e \text{ at a desired setpoint)}$$

By neglecting side forces and $\sin \alpha \approx \alpha$, $\cos \alpha \approx 1$

$$f^b \approx P_M \begin{pmatrix} 0 \\ 0 \\ K_{T_M} w_e^2 \end{pmatrix} + R^T \begin{pmatrix} 0 \\ 0 \\ Mg \end{pmatrix} \quad \tau^b \approx A(T_M)v + B(T_M)$$

$$M \begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = \begin{pmatrix} C_\psi C_\theta & -S_\psi C_\phi + C_\psi S_\theta S_\phi & S_\phi S_\psi + C_\phi S_\theta C_\psi \\ S_\psi C_\theta & C_\phi C_\psi + S_\phi S_\theta S_\psi & -C_\psi S_\phi + S_\psi S_\theta C_\phi \\ -S_\theta & C_\theta S_\phi & C_\theta C_\phi \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -K_T \end{pmatrix} \omega_e^2 + \begin{pmatrix} 0 \\ 0 \\ Mg \end{pmatrix}$$

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} 1 & S_\phi T_\theta & C_\phi T_\theta \\ 0 & C_\phi & -S_\phi \\ 0 & S_\phi / C_\theta & C_\phi / C_\theta \end{pmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$

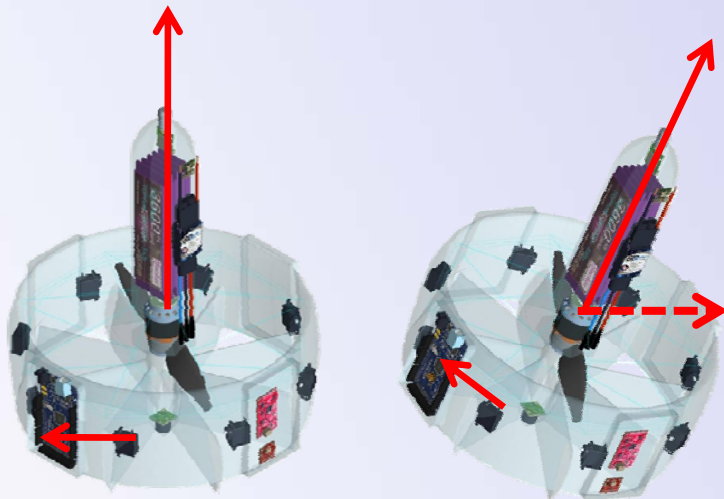
$$J \begin{pmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{pmatrix} = \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix} J \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} + \omega_e^2 \begin{pmatrix} 0 & -a_1 & 0 \\ a_1 & 0 & 0 \\ 0 & 0 & a_2 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \omega_e^2 \begin{pmatrix} 0 \\ 0 \\ K_Q \end{pmatrix} + R^T \omega_e G \bar{\omega}$$



Under-actuated system:
fewer inputs than dof

Four inputs used to control (p, ψ)

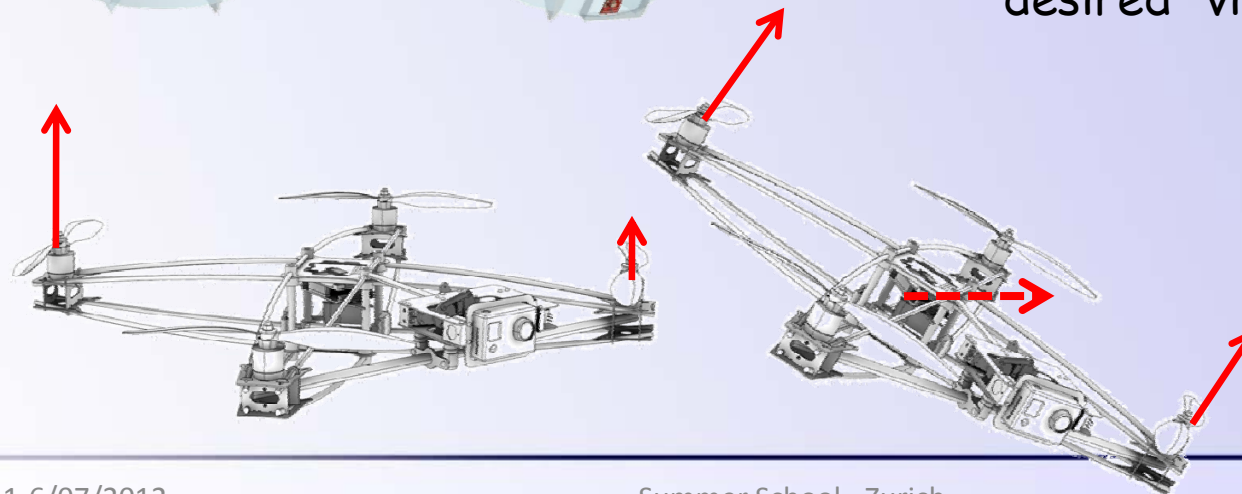
- Roll/pitch attitude as virtual input for the lateral/longitudinal dynamics



Attitude: "virtual control input"
for the lateral and long. dyn.

Force control authority:
To govern the vertical dynamics

Torque control authority:
To govern the yaw and to assign
desired "virtual control inputs"



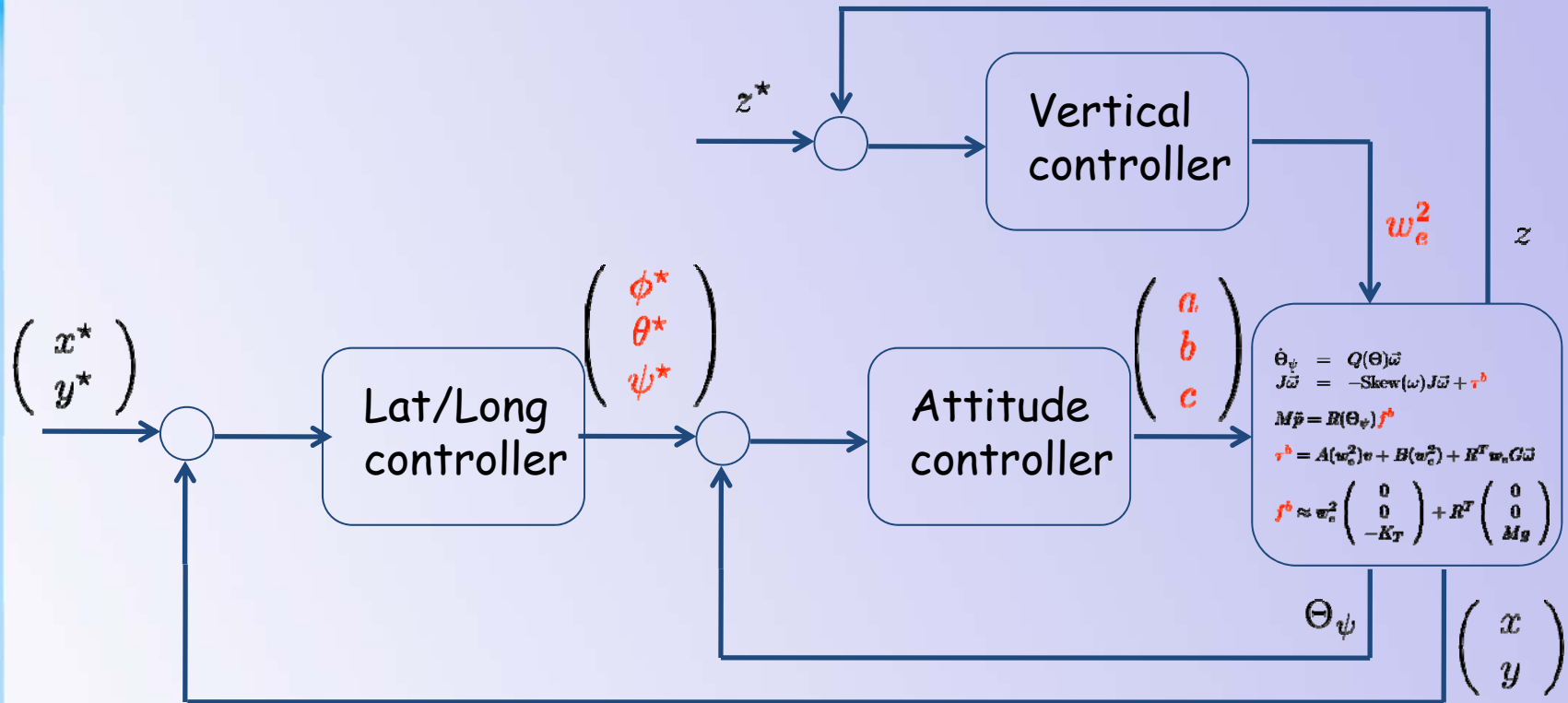
$$M \begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = \begin{pmatrix} C_\psi C_\theta & -S_\psi C_\phi + C_\psi S_\theta S_\phi & S_\phi S_\psi + C_\phi S_\theta C_\psi \\ S_\psi C_\theta & C_\phi C_\psi + S_\phi S_\theta S_\psi & -C_\psi S_\phi + S_\psi S_\theta C_\phi \\ -S_\theta & C_\theta S_\phi & C_\theta C_\phi \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -K_T \end{pmatrix} \omega_e^2 + \begin{pmatrix} 0 \\ 0 \\ Mg \end{pmatrix}$$

$$\omega_e^2 = \frac{1}{C_\theta C_\phi K_T} [Mg + \kappa_1(z - z^*) + \kappa_2 \dot{z}]$$

$$M \begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = A(\Theta_\psi) \begin{pmatrix} \tan \phi \\ \tan \theta \end{pmatrix} + B(\Theta_\psi) (\kappa_1(z - z^*) + \kappa_2 \dot{z})$$

$$A(\Theta_\psi) = \begin{pmatrix} T_\psi C_\phi / C_\theta & C_\phi \\ -C_\phi / C_\theta & T_\psi C_\phi \end{pmatrix}$$

$$\begin{pmatrix} \phi^* \\ \theta^* \end{pmatrix} = \text{atan} \left(A(\Theta_\psi)^{-1} \begin{pmatrix} \kappa_1(x - x^*) + \kappa_2 \dot{x} \\ \kappa_1(y - y^*) + \kappa_2 \dot{y} \end{pmatrix} \right)$$



Inner-Outer (cascade) control strategy

- Attitude controller (backstepping)

$$\dot{\Theta}_\psi = Q(\Theta)\omega$$

$$|\phi| < \frac{\pi}{4}, |\theta| < \frac{\pi}{4} \Rightarrow Q(\Theta) \text{ non singular}$$

$$J\dot{\omega} = -\text{Skew}(\omega)J\omega + A(w_e^2)v + B(w_e^2) + R^T w_e G \omega$$

$$\omega^* = -Q(\Theta)^{-1} \left[\kappa_\Theta (\Theta_\psi - \Theta_\psi^*) + \dot{\Theta}_\psi^* \right] \quad \begin{aligned} \tilde{\omega} &:= \omega - \omega^* \\ \tilde{\Theta}_\psi &:= \Theta_\psi - \Theta_\psi^* \end{aligned}$$

$$\dot{\tilde{\Theta}}_\psi = -\kappa_\Theta \tilde{\Theta}_\psi + Q(\theta) \tilde{\omega}$$

$$J\dot{\tilde{\omega}} = -\text{Skew}(\tilde{\omega} + \omega^*)J(\tilde{\omega} + \omega^*) + A(w_e^2)v + B(w_e^2) + R^T w_e G(\tilde{\omega} + \omega^*) - \dot{\omega}^*$$

$$v = A(w_e^2)^{-1} \left[-\kappa_\omega \tilde{\omega} + \dot{\omega}^* - B(w_e^2) - R^T w_e G(\tilde{\omega} + \omega^*) + \text{Skew}(\tilde{\omega} + \omega^*)J(\tilde{\omega} + \omega^*) \right]$$

- **Robustness?**

$$v = A_0(w_e^2)^{-1} [-\kappa_\omega \tilde{\omega} + \dot{\omega}^* - B_0(w_e^2) - R^T w_e G \omega^* + \text{Skew}(\omega^*) J_0 \omega^*]$$

with κ_ω large succeeds as long as $A(w_e^2)A_0(w_e^2)^{-1} > 0$

[Isidori, Marconi, Serrani, Springer Verlag, 2003]

- **Singularities**

- Quaternions

- Nested saturated control laws

- Linear approximation

Equilibrium point: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x^* \\ y^* \\ z^* \end{pmatrix} \quad \begin{pmatrix} \theta \\ \phi \\ \psi \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \psi^* \end{pmatrix} \quad \dot{p} = \omega = 0$

$$w_e^2 = \frac{Mg}{K_T} := w_e^{*2} \quad \begin{pmatrix} a \\ b \end{pmatrix} = 0 \quad c = -\frac{K_Q}{a_2} := c^*$$

$$\delta x = x - x^* \quad \delta y = y - y^*$$

$$\begin{pmatrix} \ddot{\delta x} \\ \ddot{\delta y} \end{pmatrix} = g \begin{pmatrix} S_{\psi^*} & -C_{\psi^*} \\ C_{\psi^*} & S_{\psi^*} \end{pmatrix} \begin{pmatrix} \phi \\ \theta \end{pmatrix}$$

$$\uparrow (\phi, \theta)$$

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \omega_x \\ \omega_y \end{pmatrix}$$

$$\uparrow (\omega_x, \omega_y)$$

$$J_{xy} \begin{pmatrix} \dot{\omega}_x \\ \dot{\omega}_y \end{pmatrix} = w_e^* \begin{pmatrix} 0 & -a_1 \\ a_1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} + I_{rot} w_e^* \begin{pmatrix} S_{\psi^*} & -C_{\psi^*} \\ C_{\psi^*} & S_{\psi^*} \end{pmatrix} \begin{pmatrix} \omega_x \\ \omega_y \end{pmatrix} \quad \leftarrow (a, b)$$

$$\delta z = z - z^*$$

$$M \ddot{\delta z} = \delta w_e^2$$

$$\delta w_e^2 = w_e^2 - w_e^{*2}$$

$$\delta \psi = \psi - \psi^*$$

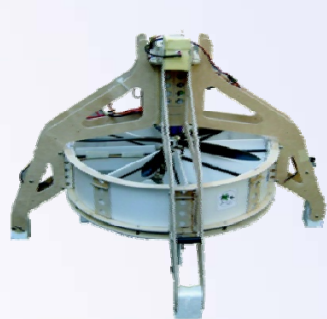
$$\dot{\delta \psi} - \omega_z$$

$$\uparrow \omega_z$$

$$J_z \dot{\omega}_z - w_e^{*2} a_2 \delta c$$

$$\uparrow \delta c = c - c^*$$

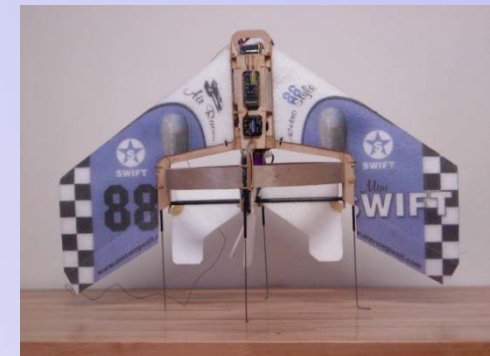
- V/STOL: Vertical/Short Take-Off and Landing
 - Combine the flight qualities of a VTOL aircraft (e.g. a helicopter) with the ones of a fixed-wing aircraft (e.g. an airplane)
 - maneuverability (VTOL)
 - flight endurance (FW)
 - hovering (VTOL)
 - high speed flight (FW)



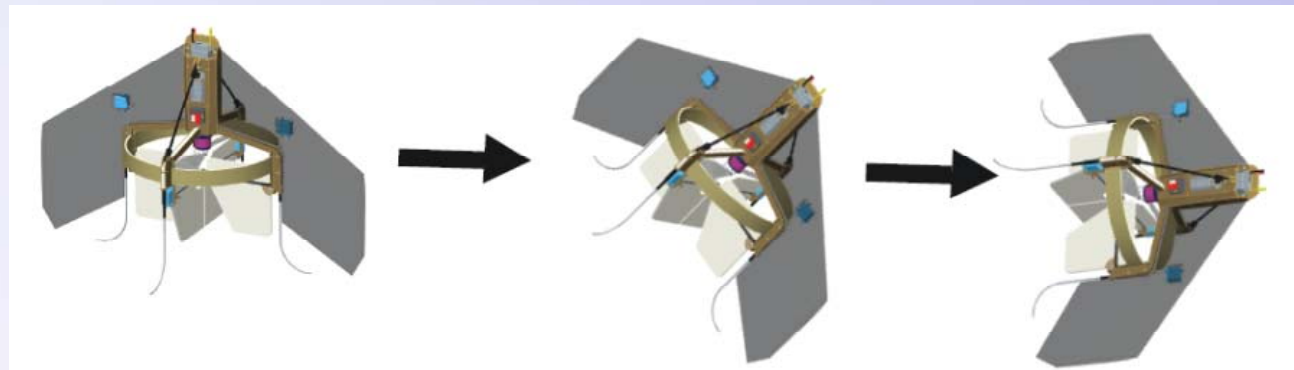
+



=



- *Goal:* change of the attitude allows one to achieve more efficient level flight starting from a hovering (low speed) flight

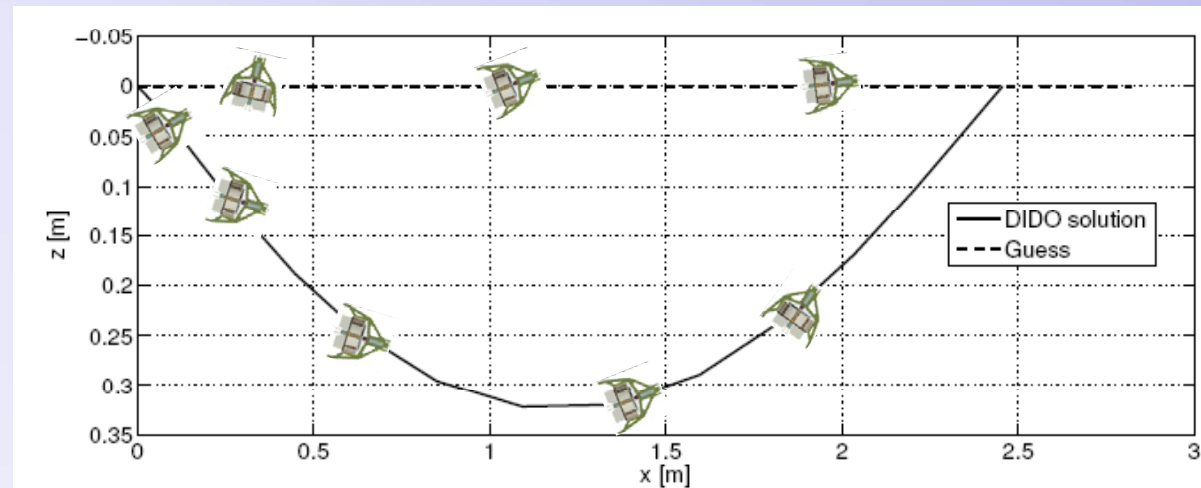


- Computation of a transition maneuver: trajectory of system state and input

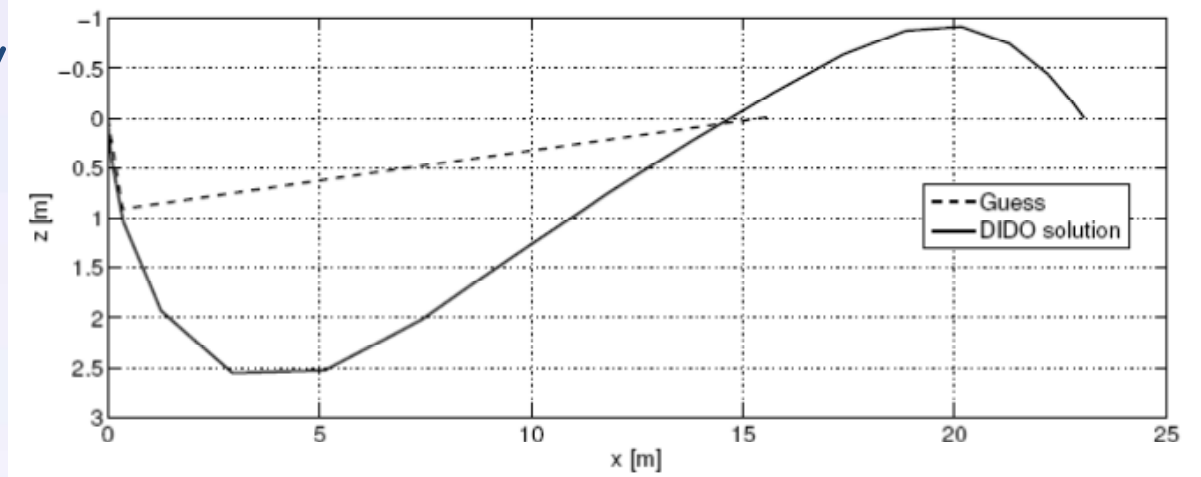
Numerical results

(R. Naldi, L. Marconi, AUTOMATICA, 2010)

Minimum time

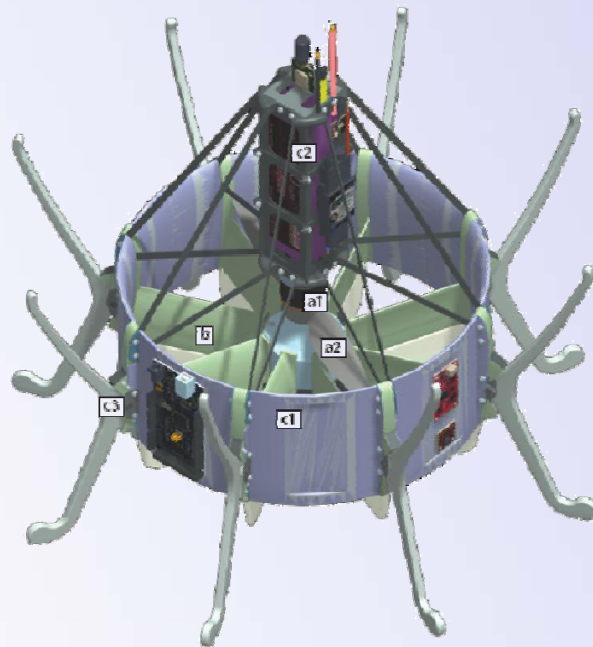


Minimum energy



- *CASY* ducted-fan UAV:

- 9 independent actuators (8 control vanes and 1 fixed pitch propeller driven by an electric motor)



Problem:

design a control allocation policy in order to obtain a desired control wrench vector

- Modular aerial robotics

