Optimizing principles underlying the shape of trajectories in goal oriented locomotion for humans

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***Abstract*— This paper addresses the problem of understanding the shape of the locomotor trajectories for a human being walking in an empty space to reach a goal defined both in position and in direction. Among all the possible trajectories reaching a given goal what are the fundamental reasons to chose one trajectory instead of another (see Fig. (1))? The underlying idea to attack this question has been to relate this problem to an optimal control problem: the trajectory is chosen according to some optimization principle. This is our basic starting assumption. The subject being viewed as a controlled system, the question becomes what criteria is optimized? Is it the time to perform the trajectory? the length of the path? the minimum jerk along the path?... In this study we show that the human locomotor trajectories are well approximated by the geodesics of a differential system minimizing the L_2 norm of the control. Such geodesics are made of arcs of clothoids. The study is based on an experimental protocol involving 7 subjects. They had to walk within a motion capture room from a fixed starting point, and to cross over distant porches from which both position in the room and orientation were changing over trials.**

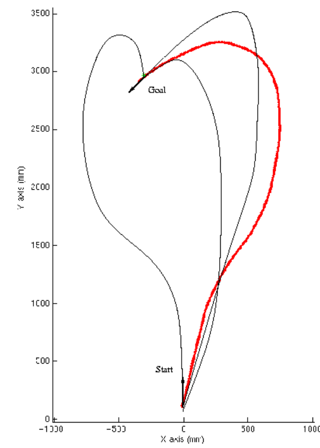


Fig. 1. Among four "possible" trajectories reaching the same goal, the subject has chosen the bold one (the other trajectories have been drawn by hand). Why?

I. Introduction

Goal-oriented locomotion has mainly been investigated with respect to how different *sensory inputs* are dynamically integrated, facilitating the elaboration of locomotor commands that allow reaching a desired body position in space. Visual, vestibular and proprioceptive inputs were analyzed during both normal and blindfolded locomotion in order to study how humans could continuously control their trajectories (see [9] and for a review, see [12]). The interaction between the relative motion of the head, the torso and the eyes has also been studied [10]. However, the principles underlying the generation (or planning) of locomotor trajectories received little attention. Recently, it has been observed that for predefined paths an inverse relationship between path geometry (curvature profile) and body kinematics (walking speed) exists [13], [25]. This empirical relation known as *"the power law"* was previously observed in [14] for drawing and hand-writing movements. In particular, the so called one third power law, was consistently reported in different experimental conditions even if no physical reason relating speed and curvature exists (see [13] for a review). Moreover, other studies suggest that the power law seems to be a by-product of a more complex behavior [19], [21]. Recent approaches based on optimization theory provide optimality principles such as total energy expended, smoothness, duration, accuracy, that encode a cost function to be minimized (for a review, see [22]). These methods are used to predict optimal movements by searching the control law according to some performance criterion.

Different hypothesis, like the maximization of the smoothness [7], [21], have been used for characterizing the production of motor behavior. In [24] the authors proposed a modified minimum jerk model accounting for the two-thirds power law. The minimum torque-change model [23] is also a smoothness optimization principle in which dynamics is involved. This model predicts also straight-line hand trajectories that correspond to the hand trajectories observed experimentally. The minimum variance model has been proposed for both, eye and arm movements [11]. This model suggests that the neural control signal is corrupted by noise. The predicted velocity profiles of eye and arm trajectories account for the speed-accuracy compromise stated by the Fitts' law [8]. Even if these models capture many aspects of observed hand trajectories and the minimum variance model also predicts eye trajectories, they have not been applied, in our knowledge, for locomotor planning.

The point of view addressed in this paper differs from the previous ones. We do not consider neither the sensory inputs nor the complexity of mechanical system modeling the human body. The point of view is complementary and more macroscopic than the standard biomechanics approaches. We rely on the observation of the geometric shape of the locomotor trajectories in the simple 3-dimensional space of both the position and the orientation of the body to compute numerically the accessibility domain of a control system. The resulting trajectories are the geodesics of the system that we try to identify according to some optimization principle. This problem is related to optimal control theory (e.g. [20]) already successfully applied to mobile robotics (e.g. [15]). In the first section we present the global methodology we followed. It consists in two major stages. The first one is to obtain a control model of the human locomotion validated by an experimental protocol involving 7 subjects walking in a motion capture room. This study has been previously published in [1]. The current paper focuses on the second stage: how to explain the shape of trajectories via optimal control.

II. METHODOLOGY

Our approach has been conducted with the following methodology:

1) The first problem is how to model the system: what is the differential controlled system that accounts for the human locomotion at best? what experimental protocol validates the model? The first stage of this work has been based on a simple statement saying that “the best way to walk is to put a foot *in front of* the other one and to start again”. “*In front of*” means that the direction of the motion is given by the direction of the body. There is a coupling between the direction of the body and the tangent to the trajectory to be realized. This is a differential non integrable coupling known as being nonholonomic[1]. The first part of this research has been to prove this statement and to provide a first control system that accounts for the locomotor trajectories. We follow a methodology based on a geometric study of the accessibility domain of the forward locomotor trajectories:

First of all we stated the problem within the 3-dimensional space of body position and direction, giving rise to the question illustrated in Fig. (1). We restrict the study to the “natural” forward locomotion with nominal speed. The model we study should be valid for all possible intentional goals reachable by a forward walk[2]. We exclude from the study the goals located behind the starting position and the goals requiring side walk steps[3]. Then we defined an experimental protocol accounting for the intentional trajectories whose goals are defined both, in position and direction. Because the objective was to cover at best the 3-dimensional accessibility region, we sampled the domain with 480 points defined by 40 positions on a 2-dimensional grid (within a 5m by 9m rectangle) and 12 directions each. The starting position was always the same. One subject performed all the 480 trajectories while other 6 performed only a subset of them chosen at random. Subjects walked from the same initial configuration to a randomly selected final configuration. The target consisted in a porch which could be rotated around a fixed point to indicate the desired final orientation (see Fig. (2)). They were instructed to freely cross over this porch without any spatial constraints relative to the path they might take. They were allowed to choose their natural walking speed to perform the task. We used motion capture technology to record more than 1400 real trajectories (see Fig. (3)). Subjects were equipped with 34 light reflective markers located on their bodies. This is the data basis used for statistical analysis and validation of the proposed models. Among the markers directly used for the analysis, the torso position (middle point (x_T, y_T) between the left and the right shoulders) and direction φ_T were found to obey a simple nonholonomic system given by (see Fig. (4)):

$$\begin{pmatrix} \dot{x_T} \\ \dot{y_T} \\ \dot{\varphi_T} \end{pmatrix} = \begin{pmatrix} \cos\varphi_T \\ \sin\varphi_T \\ 0 \end{pmatrix} u_1 + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u_2 \qquad (1)$$

where the control inputs u_1 and u_2 are the linear and angular velocities respectively. It is known that the following equation

$$\dot{y_T}\cos\varphi_T - \dot{x_T}\sin\varphi_T = 0 \qquad (2)$$

defines a non integrable 2-dimensional distribution in the 3-dimensional manifold $R^2 \times S^1$ gathering all the configurations (x_T, y_T, φ_T): the coupling between the position and the direction is said to be a nonholonomic constraint. Both linear and angular velocities appear as the only two controls that perfectly define the shape of the paths in the 3-dimensional manifold $R^2 \times S^1$.

We then used the torso trajectories for the second stage of our analysis.

2) The second part addresses the following question: given a control system, the reachable space and optimal trajectories, which is the optimal criterion that optimizes the steering of the system? Not only the question is the opposite of the classical optimal control problem (i.e. what are the trajectories which

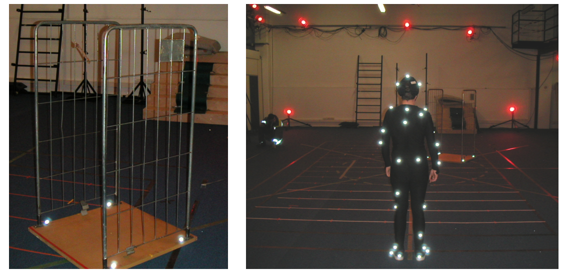
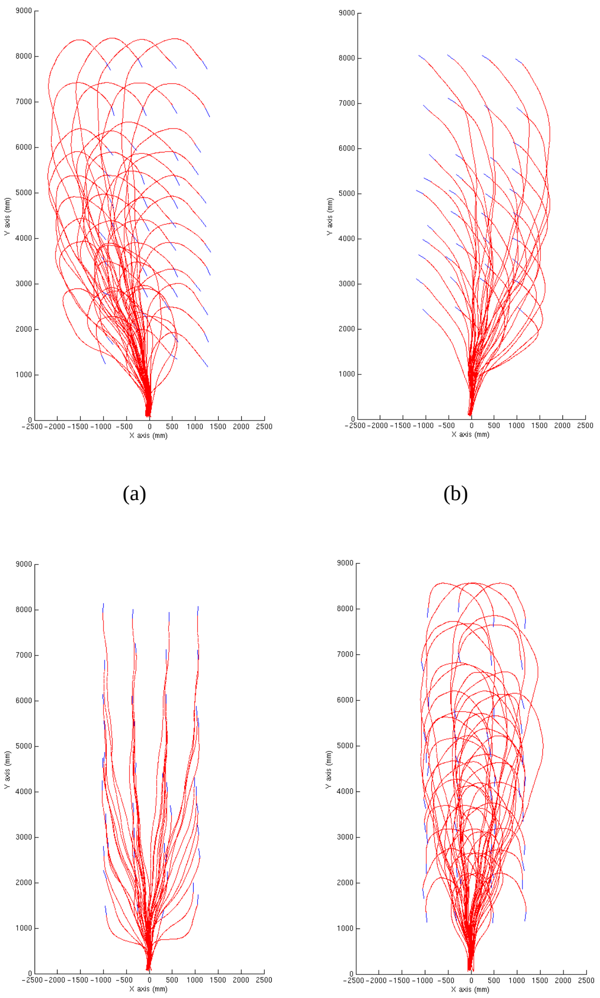


Fig. 2. The porch and the room used in the experiments.

[1]Nonholonomy is a classical concept from mechanics which has been very fruitful in mobile robotics in the past twenty years.

[2]In an empty space any goal, even located behind the starting position may be reachable by a forward walking. However this is not the “natural” way to do so.

[3]This is an important assumption: it is related to the accessibility space of a control system. Here we reasonably assume that the accessibility domain of the forward locomotion is a kind of a 3-dimensional cone approximated by the accessibility domain we consider in the protocol. Drawing the “exact” frontiers of the forward locomotion accessibility domain is typically a topic for future work open by this study.



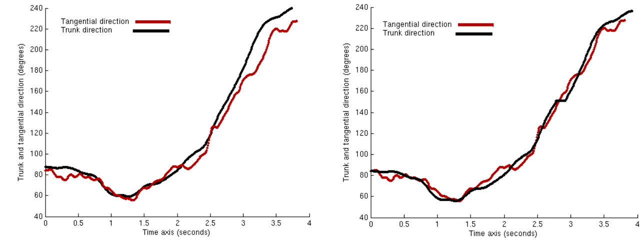
(a) (b)

(c) (d)

Fig. 3. Some examples of real trajectories with the same final orientation. (a), (b), (c) and (d) show all real trajectories where the final orientation is 330 deg., 120 deg., 90 deg. and 270 deg. respectively.

optimize a given criteria?), but it also pretends to account for a "*global*" point of view while most of the theoretical results hold only locally. Our work takes advantage of both analytical and numerical approaches to optimal control. First we apply a numerical optimization algorithm to validate the following hypothesis: locomotor trajectories are the optimal solutions of a dynamic extension of a simple unicycle control model. The validation method consists in comparing the optimal trajectories of the system with the trajectories of the data basis. The model being validated we then apply analytical methods to characterize locally the geometric shape of the geodesics. As a conclusion of this study, it is proven that:

- **the locomotor trajectories minimize the variation of the curvature, and**
- **the locomotor trajectories are well approximated by clothoid arcs.**



(a) (b)

Fig. 4. Torso direction profile with respect to the tangential direction respectively. (a) shows the torso and the tangential directions. (b) shows the torso direction shifted $\frac{1}{6}$s backward and the tangential direction. All of them correspond to the same motion (see [1]).

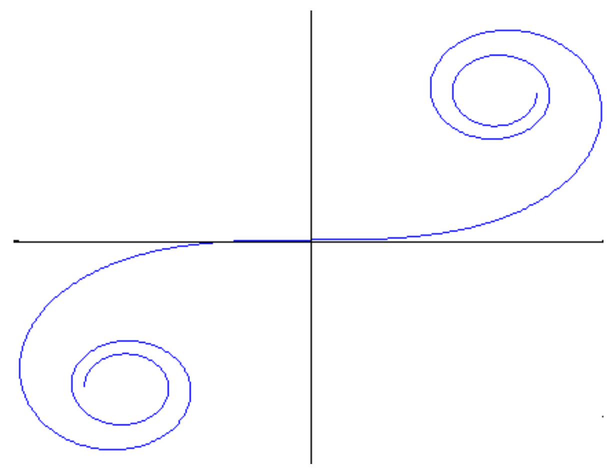


Fig. 5. Cornu spiral

A clothoid, also known as a Cornu spiral, is a curve along which the curvature κ depends linearly on the arc length and varies continuously from $-\infty$ to $+\infty$. Their equation is $\kappa = \kappa_c s + \kappa_0$ where s is the arc length, κ_0 the initial curvature and κ_c a constant characterizing the shape of the clothoid (see Fig. (5)).

III. Understanding the geometric shape of locomotor trajectories via optimal control

A. Introduction: optimal control tools

The question we address in this part of the study is to find which should be the criterion to be optimized given the experimental data and a model [16], [17]. It is the opposite optimal control problem: given a control system and the optimal trajectories, which is the performance criterion to be optimized? Unfortunately, it is not evident to answer this question in the context of motor control even if it could be more useful. Nevertheless, some tools of optimal control theory are still useful to characterize optimal trajectories minimizing different cost functions to predict locomotor trajectories verifying the nonholonomic constraints. Here we introduce a dynamic extension of system (1).

B. Model and algorithm

1) The unicycle system with inertial control law: it is noted in the previous system (1) that the curvature κ_T cannot vary continuously along a given trajectory. In order to make the curvature a variable of the system, a dynamic extension is proposed by controlling the variation of the curvature instead of the angular velocity.

$$\begin{pmatrix} \dot{x_T} \\ \dot{y_T} \\ \dot{\varphi_T} \\ \dot{\kappa_T} \end{pmatrix} = \begin{pmatrix} \cos\varphi_T \\ \sin\varphi_T \\ \kappa_T \\ 0 \end{pmatrix} u_1 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} u_2 \qquad (3)$$

The control inputs u_1 and u_2 are, for this system, the linear velocity and the variation of the curvature respectively. The nonholonomic constraint is expressed by the same Eq. (2).

2) Optimal steering of the control model: here we consider the problem of steering the System (3) from an initial state $q(0) = q_0$ to a final state $q(1) = q_f$ minimizing the L_2 norm of the control given by

$$J = \frac{1}{2}\int_0^T < (u(\tau), u(\tau)) > d\tau \qquad (4)$$

which corresponds to the least squares optimal control problem. To find a set of control inputs $u(t) \in R^m, t \in [0,T]$, which minimizes the cost J and steers the system from q_0 to q_f, we assume that the system is controllable and consequently that there exists a solution $u^* \in L_2([0,T])$ for the problem. The Hamiltonian H is defined by

$$H = \frac{1}{2}(u_1^2+u_2^2)+\psi_1\cos\varphi_T u_1+\psi_2\sin\varphi_T u_1+\psi_3\kappa_T u_1+\psi_4 u_2$$

$$\frac{\partial H}{\partial u} = 0 \begin{cases} u_1 + \psi_1\cos\varphi_T + \psi_2\sin\varphi_T + \psi_3\kappa & = 0 \\ u_2 + \psi_4 & = 0 \end{cases}$$

So we have the adjoint system $\dot{\Psi} = -\frac{\partial H}{\partial q}$ (for every $t \in [0,T]$):

$$\dot{\Psi} = \begin{cases} \dot{\psi}_1 = 0 \\ \dot{\psi}_2 = 0 \\ \dot{\psi}_3 = \psi_1\sin\varphi_T u_1 - \psi_2\cos\varphi_T u_1 \\ \dot{\psi}_4 = -\psi_3 u_1 \end{cases}$$

By differentiating the optimal controls:

$$\begin{aligned} \dot{u}_1 &= -\psi_3 u_2 \\ \dot{u}_2 &= \psi_3 u_1 \end{aligned}$$

We therefore obtain:

$$u_1^2 + u_2^2 = constant \qquad (5)$$

3) Algorithm: in general, it is difficult to find the solution of the optimal steering of nonholonomic systems, the only possibility is to rely on numerical methods[4]. We describe here the method developed by Fernandes, Gurvits, and Li [6].

Let us consider the dynamical System (3) together with the cost function given by (4). Denoting by $\{\mathbf{e}_k\}_{k=1}^{\infty}$ an orthonormal basis for $L_2([0,T])$ and considering a continuous and piecewise C^1 control law u defined over $[0,T]$, we may write a function $u \in L_2([0,T])$ in terms of a Fourier basis:

$$u = \sum_{k=1}^{\infty}(\alpha_k \mathbf{e}_k^{i\frac{2k\pi t}{T}} + \beta_k \mathbf{e}_k^{-i\frac{2k\pi t}{T}})$$

Then u can be approximated by truncating its expansion up to some rank N. The new control law u and the objective function J is then expressed as

$$u = \sum_{k=1}^{N}\alpha_k\mathbf{e}_k \Longrightarrow J \simeq \sum_{k=1}^{N} \mid \alpha_k \mid^2$$

where $\alpha = (\alpha_1, \alpha_2, ..., \alpha_N) \in R^N$ is to be determined. The configuration $q(T)$ is the solution at time T applying the control law u. Clearly, $q(T)$ appears as a function $f(\alpha)$ from R^N to R^n. In order to steer the system to q_f, an additional term must be added to the cost function:

$$J(\alpha) = \sum_{k=1}^{N} \mid \alpha_k \mid^2 + \gamma\|f(\alpha) - q_f\|^2$$

where $q(T) = f(\alpha)$ and γ is a tuning parameter during the optimization. It is proved [6] that the solution of the new finite-dimensional problem converges to the exact solution as N and γ go to infinity.

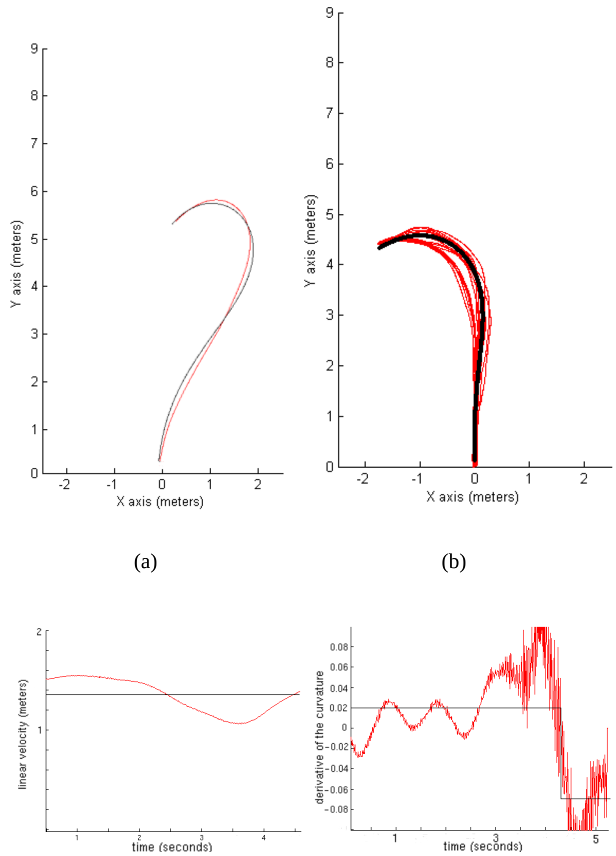
The new optimization problem becomes: given a fixed time T and q_0, q_f find $\alpha \in R^N$ such that the cost function $J(\alpha)$ is minimized. In other words, this approach will give us near-optimal paths. Because $f(\alpha)$ most of the cases is not known, we should use numerical integration to obtain $f(\alpha)$ and its Jacobian, $\frac{\partial f}{\partial \alpha} \in R^{n\times N}$. To compute a solution of the problem, it can be used a variation of the Newton's algorithm to update α.

C. Experimental results

In this section we will show the results obtained by applying the numerical optimization approach (see Section (III-B.3)) to the differential System (3) minimizing the L_2 norm of the control. It is important to emphasize that all the real trajectories have not been filtered.

We first applied the algorithm above to System (1) for all trajectories performed by the seven subjects. The results were not satisfactory. This has been the motivation to envisage the differential system with inertial control law with two control inputs: the linear velocity and the derivative of the curvature. The equation describing the system is given by (3).

[4]Few special cases similar to ours have been solved analytically: they deal with the computation of the shortest paths for Dubins's car [5], Reeds and Shepp's car [18], and some extensions of them [4], [2], [3].

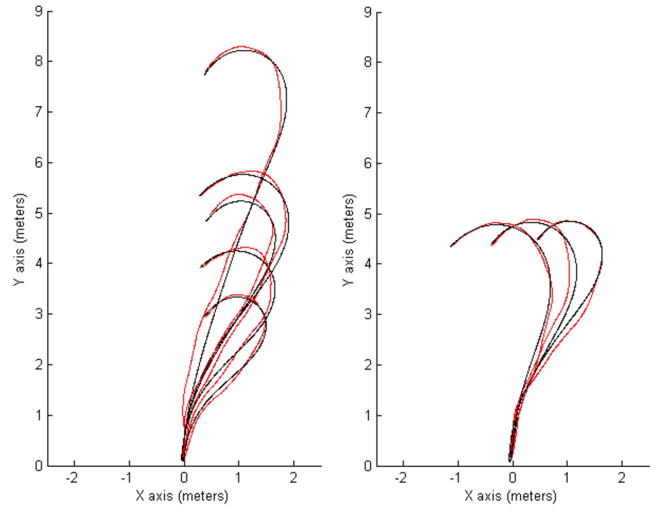


(a) (b)

(c) (d)

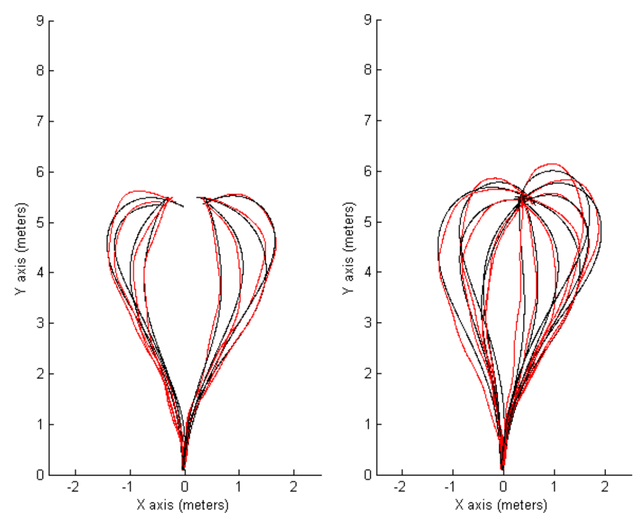
Fig. 6. Representative examples of comparisons between real (thin) and predicted (bold) locomotor trajectories. (a) shows an real trajectory performed by the first subject (thin) and the predicted optimal control-effort trajectory linking the same initial and final configurations (bold). (b) shows the real (thin) trajectories performed by the seven subjects with respect to predicted (bold) optimal control-effort trajectory. All these trajectories correspond to the same initial and final configuration. It illustrates the variability pattern and the predicted average trajectory. (c) and (d) show the comparison between the control inputs extracted from the real trajectory (filtered) of (a) and the computed optimal control inputs extracted from optimal control-effort trajectories.

To validate the model, we compared the predicted trajectories to the recorded ones performed by the seven subjects. The cost function considered was the control-effort expended. Fig. (6) shows a representative real trajectory performed by the first subject and the optimal control-effort trajectory linking the same initial and the final configurations. The real trajectory has been filtered to illustrate the comparison between the control inputs extracted from the real (filtered) trajectory and the computed optimal control inputs. Fig. (7) shows some examples of the behavior of the real and predicted trajectories by translating the final position over both: the vertical and the horizontal axes with a fixed final orientation. Fig. (8) shows some examples of real and predicted trajectories for a fixed final position. The final orientation varies in intervals of $\frac{\pi}{6}$. To measure how well



(a) (b)

Fig. 7. Representative examples of comparisons between real (thin) and predicted (bold) locomotor trajectories. (a) shows the behavior of the real and predicted trajectories by translating the final position in the vertical axis with a fixed final orientation. (b) shows the behavior of the real and predicted trajectories by translating the final position in the horizontal axis with a fixed final orientation.



(a) (b)

Fig. 8. Representative examples of comparisons between real (thin) and predicted (bold) locomotor trajectories. (a) shows symmetric real and predicted trajectories for a fixed final position. The final orientation varies in intervals of $\frac{\pi}{6}$. (b) illustrates the decision processes of the natural question: should I reach the goal by the left or by the right side?

the model predicts locomotor trajectories, we calculated for each pair of real and predicted trajectories, that correspond to the same initial and final configuration, the distance error point by point between the pair of such trajectories. Then, we computed the mean distance error dividing the sum of the errors by the number of points. After that, we compared all these errors with an arbitrary threshold of 10cm. So, for each trajectory, if the error associated is $<$ 10cm then the pair is marked as a good prediction. This procedure has been executed for 1,430 trajectories performed by seven subjects. It is interesting to note that the model approximates 90 percent of trajectories with a precision error $<$10cm. Consequently, this study proves that:

- **the locomotor trajectories are well approximated by the optimal solutions of a dynamic extension of a simple unicycle model, and**
- **the locomotor trajectories minimize the variation of the curvature.**

The statistical analysis shows that $u_1 \equiv constant$ over the whole interval of time. According to Eq. (5) we can deduce that $u_2 \equiv \beta^i$ should be a piecewise constant function. Since $\dot{\kappa}_T = u_2$ and considering that $\beta^i = \kappa_c u_1$, by integration we obtain that $\kappa_T = \kappa_c s + \kappa_{i-1}$ where κ_{i-1} is the initial curvature, $s = u_1 t$ and κ_c a constant characterizing the shape of the clothoid, therefore:

- **clothoid arcs are a good approximation of locomotor trajectories.**

IV. Conclusion

In the first part of this study, we have shown that the forward human locomotion, represented by the torso position and direction, obeys the motion of a nonholonomic system with linear and angular velocity inputs. In the second part, we were able to predict more than 90 percent of the 1430 trajectories recorded in 7 subjects during walking tasks with a $<$10 cm accuracy. We have implemented a numerical optimization algorithm to validate that the locomotor trajectories are well approximated by the optimal solutions of a dynamic extension of the unicycle model minimizing the variation of the curvature. We have locally characterized the geometric shape of the geodesics by using an analytical optimal control approach. However, the number of concatenated arcs of clothoids (switching points) are under study. Based on the data basis we have identified no more than two arcs of clothoids. As a consequence of this study, the wave-front of the locomotion may be computed.

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